Foundation Science
Physics

For Class 9

H C Verma

BHARATI BHAWAN (P & D)

Foundation Science
P H Y S I C S

FOR CLASS 9

H C Verma, PhD

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CONTENTS

1 Measurements

INTRODUCTION

Measurements are a part of our lives. When you brush your teeth, you spread a measured amount of toothpaste on the brush. You make the measurement by simply looking at the amount of toothpaste that comes out of the tube. You take almost the same amount every day because you always measure it in the same fashion. Your mother 'measures' sugar using a teaspoon before putting it in your milk. She is able to add the same amount of sugar every day because she always 'measures' the sugar in the same way. When you buy vegetables, the shopkeeper measures the mass of the vegetables using his balance and standard weights. Every period in your school has a fixed duration, which is measured using a clock.

Everybody makes measurements of some kind or the other. However, the accuracy of measurement varies from case to case. Scientists at environmental laboratories check the amount of arsenic in water to determine if it is suitable for drinking. This amount should be less than 0.00005 gram per litre of water. This measurement has to be very precise as it would affect thousands of people living in a locality. Similarly, in sports like swimming and racing, time has to be measured with an accuracy of 0.01 s. For scientific purposes, measurements have to be very accurate, and the measured values have to be stated in a precise manner. Saying that you have taken one *chullu* of water (hollowed hand to hold water is called a *chullu* in Hindi) does not describe a scientific measurement. Your chullu may not contain equal amounts of water every day. Also, different people's chullus will contain different amounts of water. Therefore, it is not a precise way of measuring volume.

A very systematic procedure is needed to get precise information about the quantity being measured. Also, scientists all over the world need to express the results of their measurements using a uniform method so that they understand each other. In this chapter we shall learn about the system developed to ensure that these requirements are met.

UNITS

Measurements are made to get information about a variety of quantities. Length is one such quantity. One may wish to measure the length of an ant, a bacterium, a table, a room, or the diameter of a tennis ball, the distance between the earth and the sun, etc. We say that length is a physical quantity. Volume, time, speed, heat, temperature, electric current, etc., are some other examples of physical quantities.

We need to define a unit for the measurement of each physical quantity. This means that we have to select a particular value (amount) of a physical quantity and say that this amount is one unit of the quantity in question. Measurement of a given quantity involves finding out the number of times of the unit the given quantity is. When we state the result of a measurement, we give this number as well the unit.

Suppose you want to measure the length of a fish you caught and tell your friends how long it is. You select a unit of length that is known to your friends. The centimetre is one such unit. A

particular length has been chosen and called one centimetre. People all over the world know how much this length is. *Centimetre is thus the name of a unit length.* Measuring the length of the fish involves finding out how many times of a centimetre the length of the fish is. If this number is 25.4, the length of the fish is 25.4 cm. One can choose a different length and call it a unit length. Of course it must be given a different name. The inch is one such unit. You can find out how many times of this unit length (i.e., inch) the length of your fish is. It will be 10 times. So, the length of the fish can also be said to be 10 inches. We then have

10 inches = 25.4 centimetres.

Similarly, we have units for other quantities such as mass, time and electric current.To avoid confusion, we should have a single, well-defined unit for each physical quantity. This need was first felt by scientists. Their efforts led to the development of a system of units called SI.

SI UNITS

The system of units that scientists use in most of their work is called *Système International d'Unités* (International System of Units), abbreviated as SI units.

There are seven basic units in the International System of Units. These are the metre (length), kilogram (mass), second (time), ampere (electric current), kelvin (temperature), candela (luminous intensity) and mole (amount of substance). These are called base units because all other units can be derived from these units.

Base and Derived Units

Suppose the unit of length has been defined. From this we can derive a unit of area. The area of a square with side equal to unit length may be called unit area. If the unit of length is taken as the metre, the unit of area will be $1\ \mathrm{m} \times 1\ \mathrm{m} = 1\ \mathrm{m}^2$. Thus, m^2, read as metre square, is the unit of area. This unit is derived from the unit of length. Similarly, the unit of volume is m^3, which is derived from the unit of length.

Take speed as another example. It measures how fast a body is moving, that is, how much distance the body covers in a given time. If we have a unit for length and a unit for time, we can derive the unit for speed from these. The speed of a body which covers unit distance in unit time may be called unit speed. Taking the metre and the second as the units for length and time respectively, unit speed would be the speed of a body that covers one metre in one second. This unit is written as m/s, read as 'metres per second'.

Can you derive the unit of mass from the units of length and time? You cannot. Length, mass and time are independent quantities. None of them can be expressed in terms of the other two. It turns out that there are only seven independent quantities, and all other quantities can be derived from these. These seven are called base quantities or fundamental quantities, and the rest are called derived quantities.

Base, or fundamental, quantities are a group of physical quantities that are independent of each other and are such that all other physical quantities can be derived from them.

The units of base quantities are called base units and those of derived quantities are called derived units. The seven base quantities and their SI units are given in Table 1.1.

Derived units are formed using the base units. Symbols for some derived units follow from the symbols used for the base units. For example, the unit of speed is m/s (metres per second). Some derived units have special names and symbols. The unit of force, for example, is given a special name—newton. The symbol for the newton is N. Similarly, the SI unit of energy is the joule, and its symbol is J. The unit of resistance is the ohm. The symbol for this unit is Ω.

Table 1.1 SI units for base quantities

Quantity	SI unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Standard Prefixes

Sometimes, when we express a quantity in SI units, its numerical value becomes very large or very small. The radius of a nucleus is around 10^{-15} m, whereas that of the earth is about 64,00,000 m. The mass of an electron is 9.1×10^{-31} kg and that of the earth is 6×10^{24} kg. In SI, standard prefixes are used for certain powers of 10. Table 1.2 lists these prefixes. You are already familiar with some of them such as milli-, kilo- and centi-. We use these prefixes together with units to make the numerical values of quantities more 'manageable'. For example, instead of saying that the radius of the earth is about 64,00,000 m, we can say that it is 6,400 km.

Table 1.2 Prefixes used with SI units

Factor	Prefix	Symbol
10^{18}	exa-	E
10^{15}	peta-	P
10^{12}	tera-	T
10^{9}	giga-	G
10^{6}	mega-	M
10^{3}	kilo-	k
10^{2}	hecto-	h
10	deca-	da
10^{-1}	deci-	d
10^{-2}	centi-	c
10^{-3}	milli-	m
10^{-6}	micro-	μ
10^{-9}	nano-	n
10^{-12}	pico-	p
10^{-15}	femto-	f
10^{-18}	atto-	a

Non-SI Units

Though it is desirable that all scientific work be done using SI units or their multiples with proper prefixes, certain non-SI units are still in use. For example, some common units for length are the inch, foot, yard and mile, and a common unit for energy is the calorie. The inch is defined as exactly 2.54 cm, 1 foot is 12 inches, 1 yard is 3 ft and 1 mile is 1760 yards. The energy unit calorie is defined as exactly equal to 4.186 J.

Uncertainty in Measurement

We use certain instruments to make measurements. All instruments have some limitations. Hence, the value of the quantity obtained from the measurement by an instrument has certain uncertainty. With ordinary wrist-watches, you cannot measure time with an accuracy better than one second. Using them, a time interval may be measured as 14 s or 15 s, but not 14.6 s. Anything between 13.5 s and 14.5 s would perhaps be noted as 14 s. Thus, if the measured value is 14 s, the actual value could be 14.0 ± 0.5 s. This ± 0.5 s is the uncertainty of the measurement. When the result of a measurement is stated, the corresponding uncertainty should also be mentioned.

The Algebra of Units

Addition, subtraction, multiplication and division form the basic operations of algebra. Interestingly, units follow the usual algebraic rules of multiplication and division. For example,

$$\begin{aligned} 1\ \text{m}^3 &= (1\ \text{m})^3 = (100\ \text{cm})^3 \\ &= (100\ \text{cm})\times(100\ \text{cm})\times(100\ \text{cm}) \\ &= (100\times100\times100)(\text{cm}\times\text{cm}\times\text{cm}) = 10^6\ \text{cm}^3. \end{aligned}$$

As another example, suppose an object's volume is 1 cm^3 and its density (ρ) is 4,000 kg/m^3. What is its mass?

We have $m = \rho V$

$$= \left(4000\,\frac{\text{kg}}{\text{m}^3}\right)\times(1\ \text{cm})^3 = \left(4000\,\frac{\text{kg}}{\text{m}^3}\right)\times\left(\frac{1}{100}\,\text{m}\right)^3$$

$$= 4000\times\left(\frac{1}{100}\right)^3\left(\frac{\text{kg}}{\text{m}^3}\times\text{m}^3\right) = 0.004\ \text{kg}.$$

Notice how the m^3 in the numerator got cancelled by the m^3 in the denominator. We encourage you to use units explicitly whenever you use the numerical value of a quantity.

• SOLVED PROBLEMS •

EXAMPLE 1 **Express 1 inch in metres.**

Solution By definition, 1 inch = 2.54 cm (exactly) $= \frac{2.54}{100}$ m = 0.0254 m (exactly).

EXAMPLE 2 **If $l = 0.90$ m, $g = 10\ \text{m/s}^2$, find the value of $\sqrt{\frac{l}{g}}$.**

Solution $\sqrt{\frac{l}{g}} = \sqrt{\frac{0.90\ \text{m}}{10\ \text{m/s}^2}} = \sqrt{0.09\ \text{m}\times\frac{\text{s}^2}{\text{m}}} = 0.3$ s.

EXAMPLE 3 **The amount of energy needed to melt 1 gram of ice is 80 calories. How many joules of energy will be needed to melt 1 kg of ice?**

Solution 1 kg = 1000 g. The energy needed to melt 1 kg of ice is therefore

(80 calories) × 1000 = 80000 calories.

Now, 1 calorie = 4.186 J.

So, 80000 calories = 80000 × 4.186 J = 334880 J = 3.3488×10^5 J.

2 Describing Motion

We see different things in motion—people and vehicles move on roads, trains move on railway tracks, aeroplanes and birds fly, our teeth go up and down while we eat, the blades of a fan move when the fan is switched on (though the fan remains at the same place), raindrops fall, the sun moves from east to west as seen from the earth, and so on.

Things are in motion inside our body too. For example, blood moves through our blood vessels, the heart pumps blood and our muscles move when we work or play. We do not see these motions, but know about them from their effects.

The different kinds of motion around us appear to be very different. But they all follow the same set of rules. The motion of a body starts or changes when a force acts on it. We shall learn about the relation between force and motion in the next chapter. In this chapter we shall learn to describe the motion of a small object in mathematical terms. But first let us understand the meaning of motion more precisely.

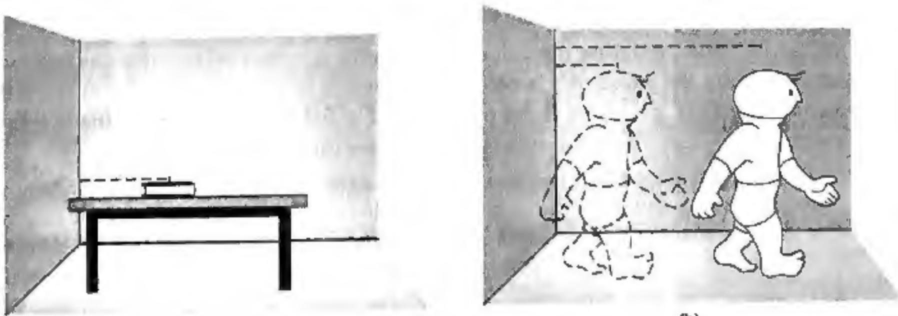
REST AND MOTION

When do we say that an object is at rest? Simple, when it is not moving. But when do we say that an object is not moving? Well, when it is at rest. But this is just like saying that the house of Mr X is opposite the house of Mr Y, and the house of Mr Y is opposite that of Mr X. We need to describe rest and motion more precisely. We define rest and motion as follows:

If the position of an object does not change as time passes, it is said to be at rest.

If the position of an object changes as time passes, it is said to be in motion.

Look at a book on your table. No one is touching your table, the fan is off and there is nothing to disturb the book. Is the book at rest or is it in motion? According to our definition, we need to find out whether or not its position changes with time. The distances of the book from the walls of the room remain unchanged. Its distances from the ceiling and the floor also remain unchanged. The position of the book remains unchanged because its distances from the walls, ceiling and floor of the room do not change. So, the book as seen from the room is at rest. We say that the book is at rest with respect to the room. If you walk inside the room, your distances from the walls change as time passes. You are in motion with respect to the room.



(a) (b)

Fig. 2.1

Motion is Relative

While sitting in a moving train, your distances from the walls, roof and floor of the compartment do not change. That is, with respect to the compartment, your position does not change. You are at rest with respect to the compartment. But your distance from the platform, from which you boarded the train, changes as time passes. So, you are moving with respect to the platform. This means that an object can be at rest with respect to one thing and in motion with respect to some other thing at the same time. So, motion is not absolute; it is relative.

Is the platform at rest, or is it moving with respect to the compartment? The distance of the platform from the compartment is changing as time passes. So, the platform is moving with respect to the compartment. Also, the compartment is moving with respect to the platform.

Suppose two trains are moving on parallel tracks in the same direction. Both started together and are moving equally fast. The distance of a person A sitting in the first train from another person B sitting in the other train does not change. So, A is at rest with respect to B. Similarly, B is at rest with respect to A. Both are moving with respect to the platform, but they are at rest with respect to each other.

POSITION OF A PARTICLE MOVING ALONG A STRAIGHT LINE

Suppose, your friend asks you the location of a shop named Liberty Tailors. Your answer may be something like this: "Liberty Tailors is on Fraser Road, at a distance of 100 metres from the Imperial Cinema, towards the south." You chose a convenient point named Imperial Cinema on Fraser Road. The location of Liberty Tailors was described by stating

(a) its distance (100 m) from the chosen point (Imperial Cinema), and

(b) the direction (south) along the road in which it is situated as seen from the chosen point.

The position of a particle (a small object) on a straight line is described in a similar way. Consider a particle moving along a straight line (Figure 2.2). To describe its position, we choose a convenient point O on the line, and we call this point the origin or reference point. We next choose the positive direction along the line. There are only two directions along the line. We call one of these positive and the other, negative. The choice is entirely ours. Either of the two directions may be called positive and the other, negative. In Figure 2.2, we have taken the left to right direction as the positive direction. Now we are ready to describe the position of the particle at any time.

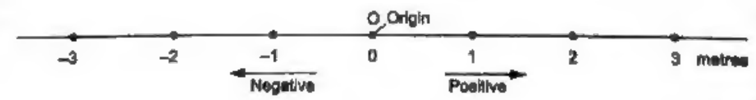


Fig. 2.2

To describe the position of the particle at a given time, we have to specify

(a) its distance from the origin, and

(b) whether it is in the positive direction or in the negative direction as seen from the origin.

This can be done by stating the value of a quantity x, which takes values as follows. If the particle is at the origin, the value of x is zero. If the particle is at a distance of 2 m from the origin in the positive direction, $x = +2$ m. If the particle is at a distance of 2 m from the origin in the negative direction, $x = -2$ m. The same rule is followed for other positions.

> The position of a particle moving along a straight line is described by a quantity x. The numerical value of x, together with the unit, gives the distance of the particle from the origin, and the sign of x denotes whether the particle is in the positive direction or in the negative direction as seen from the origin.

The quantity x itself is called the position of the particle. In real life, we often talk of motion of large objects like a bicycle, a bus, a person, and so on. These are not particles as such. But when they move through long distances, we can neglect their size and treat them as small particles.

EXAMPLE 2.1 **Consider the situation shown in Figure 2.3. (a) What is the position of a particle when it is at P_1, and when it is at P_2? (b) Are the two positions the same? (c) Are the two distances of the particle from the origin the same?**

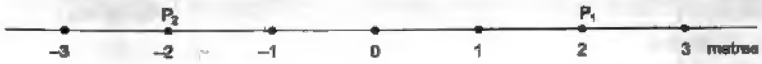


Fig. 2.3

Solution (a) The position of the particle is $x = 2$ m when it is at P_1, and $x = -2$ m when it is at P_2.
(b) The two positions are not the same.
(c) The distances of the particle from the origin in the two positions are the same, and equal to 2 m.

DISTANCE TRAVERSED AND DISPLACEMENT

When a particle goes from *A* to *B*, the total length of the path taken is the distance traversed by the particle. And, the length of the straight line *AB* together with the direction of the line *AB* gives the displacement of the particle.

When a bus goes from Delhi to Haridwar, it takes several turns along the way. To find the distance covered by the bus, we have to add the distances of all the segments of the road the bus travels on. However, to find the displacement of the bus, we need to know the straight-line distance between the bus stands at Delhi and Haridwar. But that is not enough. We also need to know the direction of Haridwar as seen from (or with respect to) Delhi.

To describe the displacement of a particle from one position to another, we must state the following:

(a) how far the final position is from the initial position (straight-line distance), and
(b) the direction in which the final position is as seen from the initial position.

The straight-line distance between the initial and final positions of a particle is called the magnitude of the displacement.

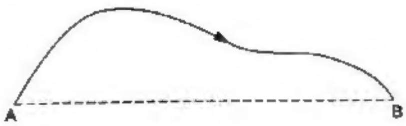


Fig. 2.4

The 'displacement' of a particle and the 'distance traversed' by it are two different quantities. Distance traversed in a given time has just a magnitude (numerical value) and no direction. On the other hand, displacement has magnitude as well as direction. Also, the magnitude of the displacement is not always the same as the distance traversed. If a particle moves in a plane or in space along a zig-zag path, the distance traversed in a given time interval may be much larger than the magnitude of its displacement in the same time interval. In Figure 2.4, a particle moves from *A* to *B* along a curve. The distance traversed is equal to the length of the curve, whereas the magnitude of the displacement is equal to the length of the straight line *AB*.

Even if the particle moves along a straight line, the distance traversed may be larger than the magnitude of its displacement. For example, if a particle goes from *A* to *B* along a straight line and then returns to *A*, the displacement is zero, but the distance traversed is not zero. Only if the particle moves along a straight line *without changing its direction*, is the magnitude of its displacement equal to the distance traversed.

EXAMPLE 2.2 **A man leaves his house at 5.30 a.m. for a morning walk and returns at 6.15 a.m. Find his displacement in this time.**

Solution The position of the man at 6.15 a.m. is the same as his position at 5.30 a.m. Thus, the distance of the final position from the initial position is zero, and hence, his displacement is zero.

Note that the distance traversed by the person in the same time is not zero unless the person kept sitting at the door for 45 minutes!

Displacement in Straight-line Motion

If a particle moves along a straight line, its displacement may be obtained from its initial position, x_1, and its final position, x_2. The displacement is given by $s = x_2 - x_1$. This gives two quantities—the straight-line distance between the initial and final positions as well as the direction of the final position as seen from the initial position. Let us understand this through an example. Consider the situation shown in Figure 2.5.

Fig. 2.5

Case I Suppose that at time t_1, the particle is at *A*, and at a later time t_2, it is at *B*.
Here $x_1 = 1$ m and $x_2 = 3$ m.
The displacement of the particle is $s = x_2 - x_1 = 3$ m $- 1$ m $= 2$ m.

Case II Suppose at time t_1, the particle is at *B*, and at time t_2, it is at *A*.
Here $x_1 = 3$ m and $x_2 = 1$ m.
The displacement is $s = x_2 - x_1 = 1$ m $- 3$ m $=$ 2 m.

Case III Suppose that the particle is at *A* at time $t = t_1$, and it is again at *A* at time $t = t_2$.
Here $x_1 = 1$ m and $x_2 = 1$ m.
The displacement is $s = x_2 - x_1 = 1$ m $- 1$ m $= 0$.

We see that displacement can be positive, negative or zero. In Figure 2.5, the positive direction is from the left to the right. If the displacement of a particle is positive, its final position is to the right of the initial position. If the displacement is negative, the final position is to the left of the initial position. If the displacement is zero, the final position is the same as the initial position. Thus, the sign of $s = x_2 - x_1$ gives the direction of the final position as seen from the initial position.

For motion along a straight line, displacement is given by a number along with a unit. The numerical value of this number tells us how far the final position is from the initial position, and the sign tells us the direction.

EXAMPLE 2.3 **The position of a particle going along a straight line is $x = 50$ m at 10.30 a.m. and $x = 55$ m at 10.35 a.m. Find the displacement between 10.30 a.m. and 10.35 a.m.**

Solution Here $x_1 = 50$ m and $x_2 = 55$ m. The displacement is $s = x_2 - x_1 = 55$ m $- 50$ m $= 5$ m.

If a particle moves in a plane or in space, the direction of the displacement cannot be given by a plus or minus sign. This is because there are an infinite number of directions from the initial position, and not just two as in the case of a straight line The displacement is then given by specifying the distance of the final position from the initial position and the direction of the final position as seen from the initial position.

SCALAR AND VECTOR QUANTITIES

Scalar Quantity

We deal with a large number of quantities in physics. Some of them may be represented by a number along with a unit. This number represents the magnitude, or the size of that quantity. The mass of a body is an example of such a quantity. The mass of a body may be 2 kg, i.e., the mass is represented by the number 2 along with the unit kg. If a body of mass 2 kg and another body of mass 3 kg are tied together, the mass of the combination is (2 + 3) kg = 5 kg. So, the values of some physical quantities (such as mass) that have only magnitude add according to the usual rules of arithmetic. A quantity that has only magnitude is called a scalar quantity. Mass, length, time, volume, density, etc., are scalar quantities.

Vector Quantity

Let us consider the displacement of a particle. It has both magnitude and direction. Suppose a particle is displaced through 1 m towards the north (*A* to *B* in Figure 2.6) and then through 1 m towards the east (*B* to *C*). What is the total displacement?

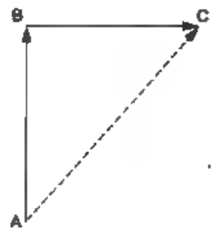


Fig. 2.6

The initial position is at *A* and the final position is at *C*. So, the total or resultant displacement is *AC*. Its magnitude $\sqrt{AB^2 + BC^2} = \sqrt{2}$ m, and its direction is 45° east of north. We see that a displacement of 1 m plus a displacement of 1 m may not be equal to a displacement of 2 m. Displacements are not added like ordinary numbers. A special rule called the triangle rule must be used for addition of displacements and certain other quantities that have magnitude as well as direction.

A quantity that has magnitude as well as direction and that follows the same rules of addition as displacements do is called a vector quantity.

Displacement is a vector quantity. We shall learn about other vector quantities like velocity, acceleration, force, etc., later in this book.

SPEED OF AN OBJECT

The speed of an object that is moving is a quantity that tells us how fast it is moving.

Average Speed

At an athletics meet, several athletes start running together and cover equal distances. The one who takes the minimum time is declared the fastest. Now suppose two athletes take part in separate races and cover different distances in different times. For example, suppose A runs 2 km in 10 minutes on Saturday and B runs 5 km in 20 minutes on Sunday. How do we decide who is faster? To decide this, we have to find the distances covered by A and B in the same time interval. If A takes 10 minutes to cover 2 km, we may say that in 1 minute he will cover

$$\frac{2\text{ km}}{10} = 0.2\text{ km}.$$

But A may not have run 0.2 km every minute during the 10 minutes he took to cover 2 km. He may have covered less than 0.2 km in some time intervals of 1 minute, and more than 0.2 km in some others. So, 0.2 km is the average distance covered in 1 minute. The overall rapidity in his 10-minute run is represented by saying that he covers *on the average* 0.2 km per minute.

Similarly, B takes 20 minutes to cover 5 km. So, in 1 minute he will cover, on the average

$$\frac{5\text{ km}}{20} = 0.25\text{ km}.$$

We therefore conclude that B runs faster than A as he covers more distance in the same time interval.

The overall rapidity of an object in a given time interval is decided by a quantity called average speed (v_{av}) which is defined as

$$\textbf{average speed} = v_{av} = \frac{\textbf{distance traversed}}{\textbf{time interval}}$$

Thus, the average speed of an object in a time interval is equal to the distance traversed divided by the time interval.

As an example, suppose that the distance from New Delhi to Patna along the railway track is 1,000 km. A train starts at 4.00 p.m. from New Delhi and reaches Patna at 8.00 a.m. the next day. Thus, it takes 16 hours to cover 1,000 km. The average speed during these 16 hours is

$$v_{av} = \frac{1000\ \text{km}}{16\ \text{h}} = 62.5\ \text{km/h}.$$

EXAMPLE 2.4 **A bus between Vishakhapatnam and Hyderabad passed the 100-km, 160-km and 220-km points at 10.30 a.m., 11.30 a.m. and 1.30 p.m. Find the average speed of the bus during each of the following intervals: (a) 10.30 a.m. to 11.30 a.m., (b) 11.30 a.m. to 1.30 p.m., and (c) 10.30 a.m. to 1.30 p.m.**

Solution

(a) The distance covered between 10.30 a.m. and 11.30 a.m. is 160 km − 100 km = 60 km. The time interval is 1 hour. The average speed during this interval is

$$v_1 = \frac{60\ \text{km}}{1\ \text{h}} = 60\ \text{km/h}.$$

(b) The distance covered between 11.30 a.m. and 1.30 p.m. is 220 km − 160 km = 60 km. The time interval is 2 hours. The average speed during this interval is

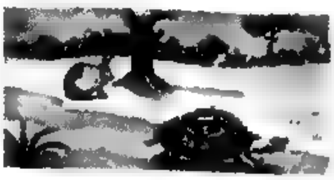
$$v_2 = \frac{60\ \text{km}}{2\ \text{h}} = 30\ \text{km/h}.$$

(c) The distance covered between 10.30 a.m. and 1.30 p.m. is 220 km − 100 km = 120 km. The time interval is 3 hours. The average speed during this interval is

$$v_3 = \frac{120\ \text{km}}{3\ \text{h}} = 40\ \text{km/h}.$$

Speed

You must have heard the story of the hare and the tortoise. The two started simultaneously from the same point for a common destination. The hare slept for some time along the way, and the tortoise reached the destination first. Since in this journey the tortoise took less time than the hare, the average speed of the tortoise was greater than that of the hare. But everyone knows that when the hare was actually running, it was much faster than the tortoise.



Average speed gives an overall idea of how fast an object moved in the whole of the time interval under consideration. How can we get an idea about how fast the object was moving at a particular instant in this time interval?

Consider a train that started at 7.00 p.m. and reached its destination at 9.00 p.m. Its average speed during the journey was 85 km/h. How fast was the train moving at 8.00 p.m.? It is a tricky question. At 8.00 p.m., the train was at a particular position, and we cannot say anything about its speed from this information. We must consider a small time interval, and find the distance covered in that interval. We can then calculate the speed by dividing the distance by the time interval. Suppose the distance covered between 8.00 p.m. and 8.01 p.m., i.e., in 1 minute was 1.5 km. The speed (v) of the train was

$$v = \frac{1.5\ \text{km}}{1\ \text{min}} = \frac{1.5\ \text{km}}{(1/60)\ \text{h}} = 1.5 \times 60\ \text{km/h} = 90\ \text{km/h}.$$

Is this the speed at 8.00 p.m.? In principle, no. This is the average speed during the interval 8.00 p.m. to 8.01 p.m. But the interval of 1 minute is quite small, and we can hope that the speed remained almost constant during this interval. The smaller the time interval, the better the approximation that the speed remains constant in that interval. We therefore define the speed of an object at time t as follows.

The speed of an object is equal to the distance traversed by it in a short time interval divided by the time interval.

We can also say that the speed of an object is the distance covered by it per unit time.

Uniform and Nonuniform Speed

If an object covers equal distances in equal time intervals, however small the intervals be, it is said to move with uniform speed.

If an object moves with a uniform (or constant) speed, its speed at any instant is the same as its average speed in any time interval. If it covers a distance s in a time interval t, its speed at any instant is

$$v = \frac{s}{t}$$

or

$$\boxed{s = vt} \qquad \ldots 2.1$$

If an object does not cover equal distances in equal time intervals, its speed is called nonuniform speed.

When a train starts, it covers a smaller distance in the first minute than the distance it covers in the next one minute. Its speed is nonuniform. After it picks up full speed, it covers equal distances every minute. Then its speed is uniform. When it approaches a station where it has to halt, it slows down. Again its speed becomes nonuniform.

EXAMPLE 2.5 **A car covers 30 km at a uniform speed of 60 km/h and the next 30 km at a uniform speed of 40 km/h. Find the total time taken.**

Solution For uniform speed,

$$s = vt.$$

If the car takes time t_1 to cover the first 30 km,

$$30 \text{ km} = (60 \text{ km/h}) \times t_1$$

or

$$t_1 = \frac{30 \text{ km}}{60 \text{ km/h}} = \frac{1}{2} \text{ h} = 30 \text{ min}.$$

Similarly, if it takes time t_2 to cover the next 30 km,

$$30 \text{ km} = (40 \text{ km/h}) \times t_2$$

or

$$t_2 = \frac{30 \text{ km}}{40 \text{ km/h}} = \frac{3}{4} \text{ h} = 45 \text{ min}.$$

The total time taken is

$$t_1 + t_2 = 30 \text{ min} + 45 \text{ min} = 75 \text{ min} = 1 \text{ h } 15 \text{ min}.$$

Unit of Speed

Speed is obtained by dividing distance by time. The unit of speed, therefore, depends on the units of distance and time. The SI unit of distance is the metre and that of time is the second. So, the SI unit of speed is metres/second. This is written in short as m/s.

If distance and time are measured in kilometres and hours respectively, the speed will be in kilometres/hour, or km/h. Conversion from one unit to another is easy. Suppose the speed of an object is 72 km/h and we wish to express it in m/s.

$$v = 72 \text{ km/h} = \frac{72 \text{ km}}{1 \text{ h}} = \frac{72 \times 1000 \text{ m}}{3600 \text{ s}} = \frac{72000}{3600} \text{ m/s} = 20 \text{ m/s}.$$

If you wish to compare two speeds, they should be represented in the same units.

EXAMPLE 2.6 **Convert 15 m/s into km/h.**

Solution

$$15\ \text{m/s} = \frac{15\ \text{m}}{1\ \text{s}} = \frac{15 \times (1/1000)\ \text{km}}{(1/3600)\ \text{h}} = \frac{15 \times 3600}{1000}\ \text{km/h} = 54\ \text{km/h}.$$

EXAMPLE 2.7 **The average speeds of a bicycle, an athlete and a car are 18 km/h, 7 m/s and 2 km/min respectively. Which of the three is the fastest and which is the slowest?**

Solution

$$18\ \text{km/h} = \frac{18\ \text{km}}{1\ \text{h}} = \frac{18000\ \text{m}}{3600\ \text{s}} = 5\ \text{m/s}.$$

$$2\ \text{km/min} = \frac{2\ \text{km}}{1\ \text{min}} = \frac{2000\ \text{m}}{60\ \text{s}} = 33.3\ \text{m/s}.$$

Thus, the average speeds of the bicycle, the athlete and the car are 5 m/s, 7 m/s and 33.3 m/s respectively. So, the car is the fastest and the bicycle is the slowest.

Typical Speeds of Some Objects

We see various objects in motion every day. Objects move with different speeds. Table 2.1 gives some typical speeds.

Table 2.1 Typical speeds

Average speed of a tortoise	0.5 km/h
Average walking speed of man	6 km/h
Maximum speed of a bee	16 km/h
Maximum speed of a cheetah	96 km/h
Maximum speed of a falcon	152 km/h
Maximum speed of a typical fast train	225 km/h
Wind speed in a light breeze	32 km/h
Wind speed in a hurricane	320 km/h
The speed of the earth moving round the sun	1.072×10^5 km/h

VELOCITY

The speed of an object tells us how fast the object is moving. If the speed is known, one can calculate the distance traversed by the object in a given time interval. To get the position of the object at the end of the interval, one needs to know the direction in which it has moved. Suppose, I tell you that at 10.30 a.m. a ball was placed at the centre of a field. A boy pushed the ball along the ground to move it with a uniform speed of 0.5 m/s. With this information can you find the location of the ball after 1 minute? You can calculate the distance covered by the ball in 1 minute.

$$s = vt = (0.5\ \text{m/s})\,(1\ \text{min}) = 0.5\,\frac{\text{m}}{\text{s}} \times 60\ \text{s} = 30\ \text{m}.$$

The ball is 30 m away from the centre of the field. But this does not tell us where exactly the ball is, i.e., 30 m from the centre in which direction. To locate the position of the ball, you need to know the direction in which the ball moved with a speed of 0.5 m/s. Let me now tell you that the boy pushed the ball towards the north. So, the ball moved with a speed of 0.5 m/s towards north. You can then locate the ball at a point 30 m north from the centre of the field.

When speed and direction are both specified, we get the velocity of the object. In the above example, we say that the velocity of the ball is '0.5 m/s, north'. This is equivalent to the statement 'the speed of the ball is 0.5 m/s, and it is moving towards the north'. We can therefore define velocity as follows.

The velocity of an object is a quantity that gives the speed of the object as well as its direction of motion.

Velocity has magnitude as well as direction. It is a vector quantity. If the speed or the direction of motion of an object is changed, its velocity changes. The velocity '5 km/h, north' is different from the velocity '7 km/h, north'. Also, the velocity '5 km/h, north' is different from the velocity '5 km/h, east'. In the first case, the directions are the same but the speeds are different, so the velocities are different. In the second case, the speeds are the same but the directions are different, so the velocities are different.

We can also define velocity as follows: the velocity of an object is the displacement of the object in a short time interval divided by the time interval

Or we can say: the velocity of an object is its displacement per unit time.

The unit of velocity is the same as that of speed. Thus, the SI unit of velocity is metres per second, written as m/s. We often use the unit km/h for convenience.

Velocity of an Object Moving along a Straight Line

When an object moves along a straight line, there are only two possible directions of motion. In such a case, its velocity may be represented in a very simple manner. We write the speed of the object, and put a plus sign before it if the object is moving in the positive direction of the line, and a minus sign if it is moving in the negative direction of the line. The resulting number gives the speed as well as the direction of the motion, and hence, represents velocity.

Uniform Velocity

If the velocity of an object does not change as time passes, it is said to move with a uniform velocity. In such a case, both its speed and direction remain constant. This means that the object is moving along a straight line, without turning back, with a fixed speed. The displacement of the particle is equal in equal time intervals, however small a time interval we choose. We also say in this situation that the object is in uniform motion If the object undergoes unequal displacements in equal time intervals, the motion is nonuniform.

EXAMPLE 2.8 **The positions of a scooter along a straight road from a reference point at different times are given in the following table. Is the motion uniform or nonuniform from 10.30 a.m. to 10.40 a.m.? From 10.35 a.m. to 10.55 a.m.?**

Time	10.30 a.m.	10.35 a.m.	10.40 a.m.	10.45 a.m.	10.50 a.m.	10.55 a.m.
Position	5.0 km	7.0 km	11.0 km	15.0 km	19.0 km	23.0 km

Solution From 10.30 a.m. to 10.35 a.m. the scooter covers 2.0 km, but from 10 35 a m. to 10.40 a.m. it covers 4.0 km. In both cases the time inverval is 5 minutes. So, the scooter has not undergone equal displacements in equal time intervals. So the motion is nonunifrom between 10.30 a.m. and 10.40 a.m.

Now look at the period 10 35 a.m. to 10.55 a.m. It is divided into time intervals of 5 minutes. The distance covered is 4.0 km in each of these intervals. The displacements are also equal. So, the motion seems to be uniform between 10 35 a.m. and 10.55 a.m.

Why do we say "seems" to be uniform? Because we do not know how exactly the scooter moved in any given period like 10.35 a.m. to 10.40 a.m. It may have moved faster than its average speed from 10.35 a.m. to 10.36 a.m., and slower from 10.36 a.m. to 10.37 a.m. So, from the given information we can only say that the motion "seems" to be uniform.

When an object moves with a uniform velocity, the displacement of the object is equal in equal intervals of time. If the velocity is v, the displacement s in a time interval t is given by

$$\boxed{s = vt}$$

Difference between Speed and Velocity

Speed is a scalar quantity while velocity is a vector. Speed of an object at a given time tells us how fast the object is moving at that time. This information is also given by its velocity. In addition, velocity also indicates the direction in which the object is moving. Thus, if velocity is given, we know the speed. But if only the speed is given, we don't know the velocity.

Whenever an object changes its direction of motion, its velocity changes. But its speed need not. Consider a boy running on a circular path. He may be running with a constant speed (equally fast all the time), but his velocity keeps changing since his direction of motion keeps changing.

The velocity of an object moving along a straight line is given by a number, which can be zero, positive or negative. But its speed can never be negative—it is simply the magnitude of its velocity. The speed and velocity of an object are given by the same number only when it moves along the positive direction of a straight line.

ACCELERATION OF AN OBJECT MOVING ALONG A STRAIGHT LINE

When a train starts from a station, its velocity increases for some time. The velocity is zero when it just starts. It is only a few kilometres per hour after about 10 seconds, so a person on the platform can manage to keep pace with the train, talking to a passenger in the train. The velocity becomes very high within a few minutes. Afterwards, when the train approaches a station where it has to stop, its velocity gradually decreases, before becoming zero. This means that the velocity of the train changes (increases or decreases) during its motion. When the velocity of an object changes with time, i.e., when the motion is nonuniform, the object is said to have an acceleration.

The rate at which the velocity of an object changes is called the acceleration of the object.

In other words, acceleration is the change in velocity per unit time.

The change in velocity may be due to change in speed or due to change in the direction of motion. First we will consider acceleration due to change in speed.

Consider an object moving along a straight path in the same direction (without turning around). Its velocity changes in a short time interval t. Its velocity at the beginning of the time interval is u (called the initial velocity). Its velocity after time t is v (called the final velocity).

The change in velocity per unit time, i.e., the acceleration is

$$a = \frac{v - u}{t} \quad \text{...2.2}$$

or

$$v = u + at \quad \text{...2.3}$$

The acceleration of an object may or may not be constant. Suppose the velocity of a scooter changes with time as given in Table 2.2.

Table 2.2 Velocity of a scooter at different instants

Time in seconds	0	1	2	3	4	5	6	7	8	9	10
Velocity in km/h	0	0.5	1.0	1.5	2.0	2.5	3.2	4.0	6.0	9.0	9.5

We see that during the time interval 0–1 s, the velocity changes from 0 to 0.5 km/h. In the equal time intervals 1–2 s, 2–3 s, 3–4 s and 4–5 s, the velocity changes by an equal amount, i.e., 0.5 km/h. So, the acceleration (the rate of change of velocity) remains the same during the interval 0–5 s. This is an example of motion with constant acceleration.

The velocity changes by 0.7 km/h during 5–6 s, 0.8 km/h during 6–7 s, 2.0 km/h during 7–8 s, 3.0 km/h during 8–9 s and 0.5 km/h during 9–10 s. So, from 5 s to 10 s, the acceleration is not constant.

If the acceleration of an object is constant in a time interval 0-t, its value can be obtained from Equation 2.2 or 2.3. However, if the acceleration varies with time, these two equations cannot be used.

EXAMPLE 2.9 **The velocity of a car at 10.50 a.m. is 60 km/h and at 10.52 a.m. it is 80 km/h. Assuming constant acceleration in the given period, find its value.**

Solution

$$a = \frac{v-u}{t} = \frac{(80\text{ km/h})-(60\text{ km/h})}{2\text{ min}} = \frac{20\text{ km/h}}{(1/30)\text{ h}} = 600\text{ km/h}^2.$$

Unit of Acceleration

Acceleration is obtained by dividing the change in velocity by time The SI unit of velocity is m/s and that of time is s. Thus, the SI unit of acceleration is (m/s)/s = (m/s)^2, read as metre per second square. In any unit of acceleration, there is a length unit in the numerator and the square of a time unit in the denominator. In Example 2.9, for instance, the acceleration is expressed in km/h^2

Acceleration Has a Direction

Like velocity, acceleration also has a direction. It is a vector quantity. For a car whose speed increases while it is moving on a straight road, the acceleration is in the direction of its motion. When you drop a ball, it falls vertically, and its speed increases Its acceleration is thus in the vertically downward direction. To describe acceleration we need to describe its direction also.

EXAMPLE 2.10 **An object is sliding down an inclined plane. The velocity changes at a constant rate from 10 cm/s to 15 cm/s in two seconds. What is its acceleration?**

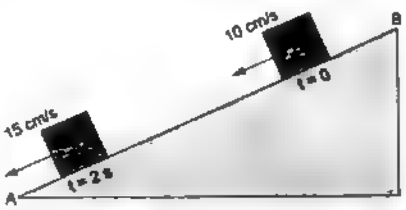


Fig. 2.7

Solution The situation is shown in Figure 2.7. Let us take BA as the positive direction. The velocity at $t=0$ is $u=+10$ cm/s and that at $t=2$ s is $v=+15$ cm/s.

Thus, $a = \frac{v-u}{t} = \frac{15\text{ cm/s} - 10\text{ cm/s}}{2\text{ s}}$ 2.5 cm/s^2.

The acceleration is positive, which means it is in the direction BA.

EXAMPLE 2.11 **An object is moving up an inclined plane. Its velocity changes from 15 cm/s to 10 cm/s in two seconds. What is its acceleration?**

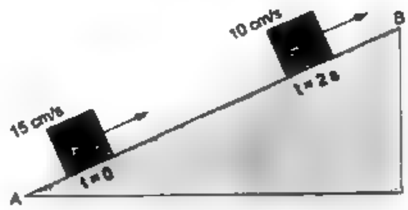


Fig. 2.8

Solution The situation is shown in Figure 2.8. Let us take AB as the positive direction. The velocity at $t=0$ is $u=+15$ cm/s and that at $t=2$ s is $v=+10$ cm/s. Thus,

$$a = \frac{v-u}{t} = \frac{10\text{ cm/s} - 15\text{ cm/s}}{2\text{ s}} = -2.5\text{ cm/s}^2.$$

The acceleration comes out to be negative. This means it is in the direction BA.

It is interesting to note that the direction of acceleration is the same (in the direction BA) in both the examples given above, though we calculated it to be + 2.5 cm/s^2 in one case and 2.5 cm/s^2 in the other This is because the choice of the positive direction was different in the two cases. The acceleration is the same—in both cases 2.5 cm/s^2 down the incline

Deceleration

If the speed of an object decreases, we say that it is decelerating, or it has a deceleration or retardation.

In Example 2.11, the *magnitude* of the object's velocity, i.e., its *speed*, decreases from 15 cm/s to 10 cm/s. Therefore, we conclude that the object was decelerating.

Just because the acceleration of an object is negative does not mean that it is decelerating. In Example 2.11 the acceleration is negative, but that is because of our choice of *AB* as the positive direction. If we choose *BA* as the positive direction, $u = -15$ cm/s and $v = -10$ cm/s. In this case, the acceleration is

$$a = \frac{v-u}{t} = \frac{-10\ \text{cm/s} - (-15\ \text{cm/s})}{2\ \text{s}} = 2.5\ \text{cm/s}^2 .$$

Although the object decelerated, the choice of the positive direction along *BA* gave us a positive value of acceleration. But, as you would have noticed, the sign of the acceleration turned out the opposite of the sign of the velocities for both choices of the positive direction. Thus, we conclude that if the acceleration's direction is opposite to that of the velocity, the object will decelerate.

EXAMPLE 2.12 **A boy throws a ball up and catches it when the ball falls back. In which part of the motion is the ball decelerating?**

Solution As the ball goes up, its speed decreases. As its comes down, its speed increases. Thus, it decelerates while going up.

GRAPHS

A graph is a very powerful method of presenting information. In newspapers, magazines and on TV you find graphs representing different information. Look at Table 2.3. It shows the score at the end of each over in a thirty-over one-day cricket match. The same information is presented in the form of a graph in Figure 2.9. Although the table and the graph contain exactly the same information, it is a lot easier to get an overall idea of how the scoring progressed from the graph. One can see at a glance from the graph that the scoring was very slow in the first 10 overs, it gradually picked up in the next 10 overs, it was again slow in the overs 20–25, and in the final 5 overs the scoring was very fast.

Table 2.3 Score at the end of each over in a cricket match

Over	Score	Over	Score	Over	Score
1	2	11	20	21	67
2	3	12	24	22	68
3	5	13	29	23	69
4	6	14	35	24	70
5	7	15	40	25	72
6	9	16	44	26	82
7	10	17	49	27	90
8	12	18	55	28	101
9	14	19	60	29	111
10	15	20	65	30	121

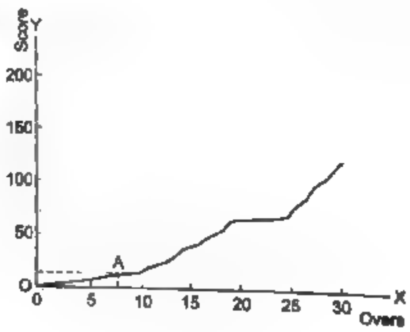


Fig. 2.9

Plotting a Graph

A graph is plotted to display the relation between two quantities. Generally, one of the two quantities changes independently and the other quantity depends on it. For example, in Table 2.2, we have two quantities—*time* and *velocity*. Time is the independent quantity and velocity

depends on it. In Table 2.3, *number of overs* is the independent quantity and *score* depends on it. To plot a graph from a table of values, the following steps are needed:

(a) **Choosing the axes** Draw two perpendicular lines, crossing each other at a point. Each line represents one of the two quantities to be plotted. Generally, a horizontal line from left to right is drawn to represent the independent quantity, and a perpendicular line is drawn to represent the dependent quantity. These lines are called the x-axis and the y-axis, or the horizontal axis and the vertical axis respectively. In Figure 2.9, the line OX represents the number of overs and the line OY represents the score. These lines are also named after the quantities they represent. Thus, in Figure 2.9, the line OX is called the *over-axis* and the line OY, the *score-axis*.

(b) **Choosing the scale** The size of the paper on which a graph is drawn is limited. On the available length of the axes, values are marked at equal distances. This is done in such a way that all the values of the quantity represented on the axis can be accommodated in the available length.

(c) **Plotting the points** Each set of values of the two quantities is represented by a point on the graph. For example, the set '8 overs, 12 runs' in Table 2.3 is represented by the point A in the graph. To get this point, we mark the point corresponding to 8 overs on the over-axis and draw a perpendicular on the over-axis at this point. Similarly, we mark the point corresponding to 12 runs on the score-axis and draw a perpendicular on the score-axis at this point. The point of intersection of these perpendiculars is the point A, representing the set '8 overs, 12 runs'.

(d) **Joining the points** Once all the points corresponding to the available sets of values of the quantities are plotted, they are joined by a smooth curve to get the graph.

Distance–Time Graph

The nature of the motion of an object can be studied by plotting a graph between the distance covered and time. Such a graph is called a distance–time graph.

Consider a car on a long journey. It moves with a constant speed, covering 50 km every 20 minutes. Table 2.4 shows the distance of the car from the starting position at different instants of time.

Table 2.4 Distance covered by the car at different instants of time

Time (hours)	Distance (km)
10.00	0
10.20	50
10.40	100
11.00	150
11.20	200
11.40	250
12.00	300
12.20	350
12.40	400
13.00	450
13.20	500

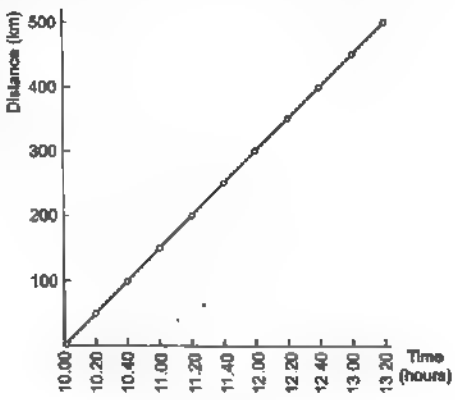


Fig. 2.10

In Figure 2.10, we have taken the distance covered on the y-axis and time on the x-axis, and plotted each set of values. The resulting graph is the distance–time graph for the car. The points on the graph fall on a straight line. This happens when an object covers equal distances in equal time intervals, i.e, when it moves with a uniform speed.

So, the distance–time graph of an object moving with a uniform speed is a straight line. Conversely, if the distance–time graph of an object is a straight line, the object is moving with a uniform speed.

Note that if the distance–time graph is a straight line, it does not mean that the object is moving along a straight line. The 500-km drive described in Table 2.4 need not be along a straight road, although the distance–time graph is a straight line.

Speed from distance–time graph

Consider an object moving with a uniform speed. The distance–time graph is represented by a straight line as shown in Figure 2.11. We can calculate the speed of the object from the distance–time graph. From the graph shown in Figure 2.11, we find that the distance covered till time t_1 is d_1 and till time t_2 it is d_2. Thus, the object covers a distance of $d_2 - d_1$ in the time interval $t_2 - t_1$. The speed of the object is, therefore,

$$v = \frac{d_2 - d_1}{t_2 - t_1} = \frac{BC}{AC}.$$

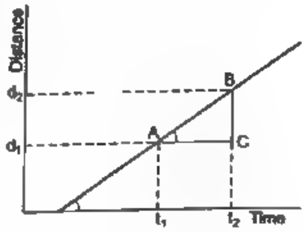


Fig. 2.11

The ratio $\frac{BC}{AC}$ is called the slope of the line. Thus, if the distance–time graph of an object is a straight line, the speed of the object is equal to the slope of the straight line.

The slope of a line tells us how steeply it is inclined to the horizontal axis (drawn from left to right). If the line is parallel to the horizontal axis, the slope is zero. As the line gets more and more inclined to this axis, its slope increases. Thus, a more steeply inclined distance–time graph indicates greater speed.

EXAMPLE 2.13 **Figure 2.12 shows distance–time graphs of two objects A and B. Which object is moving with a greater speed when both are moving?**

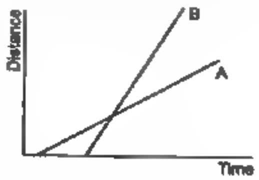


Fig. 2.12

Solution The line for object B makes a larger angle with the time-axis. Its slope is, therefore, larger than the slope of the line for object A. Thus, the speed of B is greater than that of A.

EXAMPLE 2.14 **Two friends A and B started from the same location and went 30 km along a road in the same direction. Figure 2.13 shows their motions through graphs. Answer the following questions.**

(a) When did A start?

(b) When did B start?

(c) When B started, how far away was A from B?

(d) Did any of them move with uniform speed?

(e) Which of the two had greater speed at 10.30 a.m.? Which had greater speed at 11.15 a.m.?

(f) When and where did B overtake A?

(g) How long did B wait for A after reaching the destination?

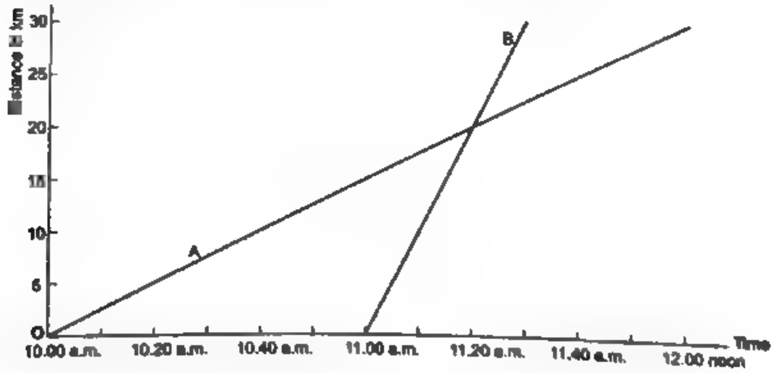


Fig. 2.13

Solution

(a) A started at 10.00 a.m., when the distance covered by him was zero.

(b) B started at 11.00 a.m., when the distance covered by him was zero.

(c) B started at 11.00 a.m. We draw a line perpendicular to the time-axis from the point 11.00 a.m. (Figure 2.14). This line cuts the graph of A at *a*. We now draw a line *ab* perpendicular to the distance-axis from *a*. This cuts the distance-axis at *b*. The value of the distance at *b* is 15 km. Thus, A had covered a distance of 15 km by 11.00 a.m. So, A was 15 km ahead of B when B started his journey.

(d) Both the graphs are straight lines. So, both of them moved with uniform speeds.

(e) At 10.30 a.m., A was moving with some speed whereas B had not yet started his journey. The speed of B was zero. Thus, the speed of A was greater than the speed of B. At 11.15 a.m., both were moving with uniform speeds. The slope of the graph of B is greater than that of the graph of A. Thus, B was moving at a greater speed.

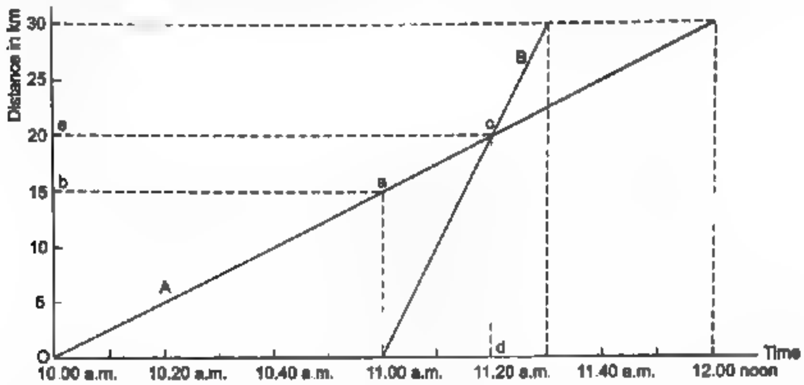


Fig. 2.14

(f) At the instant B overtook A, both were at the same place on the road. The distances covered by the two friends from the starting point were the same at this instant From Figure 2.14 we see that *c* is the point where the two distances are the same at the same time. To get this time, we draw a perpendicular *cd* from *c* to the time-axis. This cuts the time-axis at *d*. The time corresponding to this point is 11.20 a.m. To get the distance from the starting point, we draw a perpendicular *ce* from *c* to the distance-axis. It cuts the distance-axis at *e*, which corresponds to 20 km. Thus, B overtook A at 11.20 a.m. at a point 20 km away from the starting point.

(g) From the graph, we can read the time when B completed his 30-km journey. This time is 11.30 a.m. Similarly, A completed his journey of 30 km at 12.00 noon. So, B waited for 30 minutes for A, after reaching the destination.

Distance–time graph for nonuniform speed

If an object moves with a nonuniform speed, its distance–time graph is not a straight line. Figure 2.15 shows an example. The inclination of the graph is different at different places, and hence, it does not have a unique slope.

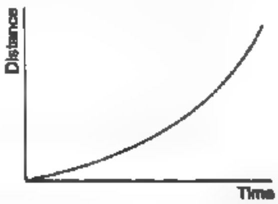


Fig. 2.15

Displacement–Time Graph

Consider an object moving along a straight line. Its displacement in a given time interval is then represented by a number. If a graph is plotted by taking the displacement on the vertical axis, and time *t* on the horizontal axis, we get the displacement–time graph of the object. The slope of the displacement– time graph of an object gives its velocity.

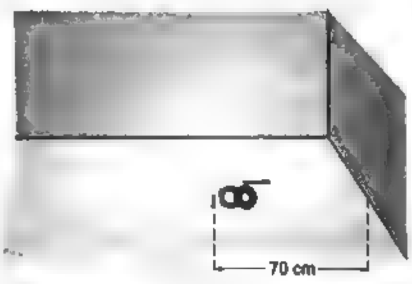


Fig. 2.16

Consider the situation in Figure 2.16. A ball placed on a smooth floor is made to move towards a wall. Table 2.5a gives the displacement of the ball from its initial position at different instants of time. The displacement versus time graph is shown in Figure 2.17 From the table we can see that the velocity of the ball is 10 cm/s.

Table 2.5a

Time in s	0	1	2	3	4	5	6	7
Displacement in cm	0	10	20	30	40	50	60	70

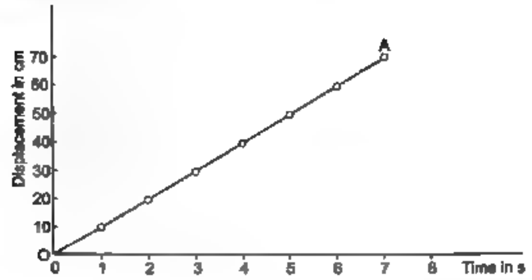


Fig. 2.17

Suppose the ball hits the wall at $t = 7$ s and returns along the same line. The displacement now decreases as the ball approaches the starting point. Table 2.5b gives the displacement of the ball from its initial position at different instants of time. It is clear from the table that the velocity of the ball is 10 cm/s.

Table 2 5b

Time in s	8	9	10	11	12	13	14
Displacement in cm	60	50	40	30	20	10	0

Let us extend the graph of Figure 2.17 up to $t = 14$ s. The full graph from $t = 0$ to $t = 14$ s is shown in Figure 2.18. Note that the line makes an acute angle with the time-axis when the velocity is positive (during 0–7 s). And it makes an obtuse angle with the time-axis when the velocity is negative (during 7–14 s).

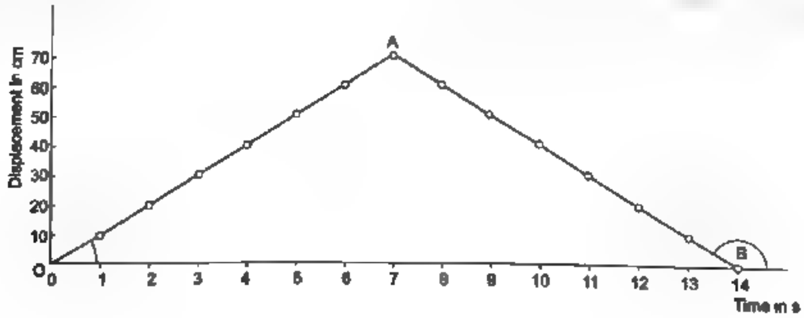


Fig. 2.18

Speed–Time Graph

Some information about the motion of an object can also be obtained from its speed–time graph.

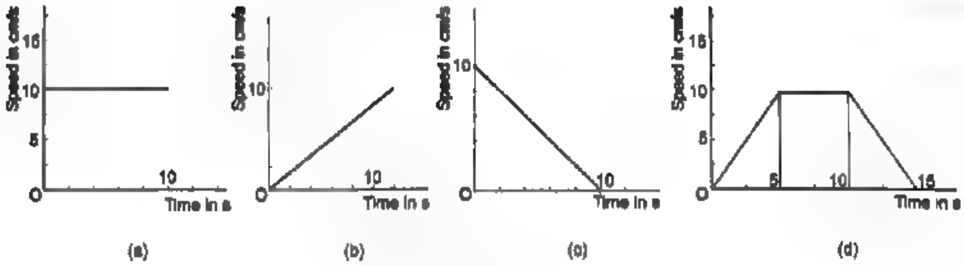


Fig. 2.19

Figure 2.19 gives the speed–time graphs of four different objects in motion. Let us see what information can be obtained from these graphs. Figure 2.19a shows that the speed does not change with time, i.e., the speed has the same value (10 cm/s). Thus, it represents an object moving with a constant speed.

Whenever an object moves with a constant speed, its speed–time graph is a straight line, parallel to the time-axis.

Figure 2.19b shows that the speed continuously increases with time. At time $t = 0$, the speed is 0. At $t = 10$ s, it becomes 10 cm/s. The straight-line nature of the graph indicates that the speed increases at a constant rate.

Figure 2.19c shows that the speed is 10 cm/s at $t = 0$ and gradually decreases as time passes. Thus it represents a decelerating object. Here also the speed changes at a constant rate. At $t = 10$ s, the speed becomes zero.

Figure 2.19d represents the motion of an object which speeds up from $t = 0$ to $t = 5$ s, then moves at a constant speed from $t = 5$ s to 10 s and then decelerates to a stop at $t = 15$ s.

Calculation of distance from the speed–time graph

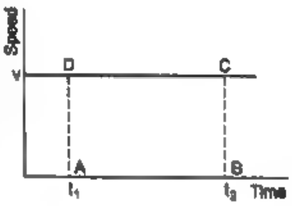


Fig. 2.20

A. Uniform speed Suppose an object is moving with a uniform speed v. The distance covered by this object during a time interval t_1 to t_2 is $s = v(t_2 - t_1)$.

Figure 2.20 shows the speed–time graph. We have $AD = BC = v$ and $AB = t_2 - t_1$. Thus the distance covered is

$$s = v(t_2 - t_1) = AD \cdot AB$$
$$= \text{area of the rectangle } ABCD.$$

The method for finding the distance covered in a time interval t_1 to t_2 using a speed–time graph is as follows.

Draw perpendicular lines to the time-axis at the points A and B corresponding to t_1 and t_2. The area enclosed by these perpendicular lines, the time-axis and the speed–time graph is equal to the distance covered in the time interval t_1 to t_2.

This area is often called the area under the graph. We have stated that the area $ABCD$ is equal to the distance covered, s. But can we equate an area to a distance? Certainly not. An area is measured in square metres and distance is measured in metres. They cannot be equated. But the 'area under a graph' is not an 'area' in the usual sense of the word. This is because the lengths along the axes are not the usual length. Normally, length is measured in metres. But the length AB along the time-axis is measured in seconds and the length AD along the speed-axis is measured in metre/second. That is why the product $AD \cdot AB$ is in metres.

$$\frac{\text{metre}}{\text{second}} \times \text{second} = \text{metre}.$$

EXAMPLE 2.15 **Figure 2.21a represents the speed–time graph for a particle. Find the distance covered by the particle between $t = 10$ min and $t = 30$ min.**

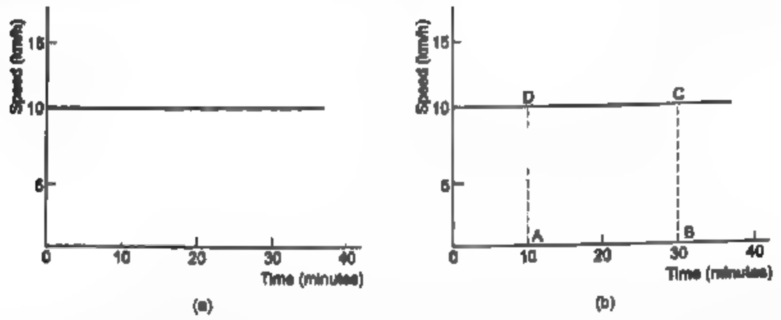


Fig. 2.21

Solution We draw perpendicular lines from the 10-minute point and the 30-minute point to the time-axis (Figure 2.21b). The distance covered is equal to the area of the rectangle $ABCD$. Its value is

$$AB \cdot AD = (30 \text{ min} - 10 \text{ min}) \times (10 \text{ km/h})$$
$$= 20 \text{ min} \times 10 \text{ km/h}$$
$$= \frac{20}{60} \text{ h} \times 10 \text{ km/h} = \frac{10}{3} \text{ km}.$$

B. Nonuniform speed The method discussed above to calculate the distance covered from a speed–time graph assumes that the object moves with a uniform speed. However, the method is equally applicable if the object moves with nonuniform speed. Figure 2.22 shows a speed–time graph for an object moving with nonuniform speed. Let us use the above method to get the distance covered in the time interval t_1 to t_2. The points A and B on the time-axis correspond to the times t_1 and t_2 respectively. We draw perpendiculars AD and BC on the time-axis from A and B respectively. These perpendiculars meet the graph at D and C. The area $ABCD$, shown shaded in the figure, represents the distance covered in the time interval t_1 to t_2.

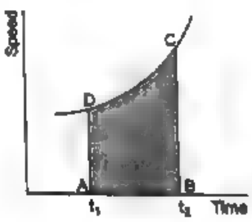


Fig. 2.22

EXAMPLE 2.16 **Find the distance covered by a particle during the time interval $t = 0$ to $t = 20$ s for which the speed–time graph is shown in Figure 2.23.**

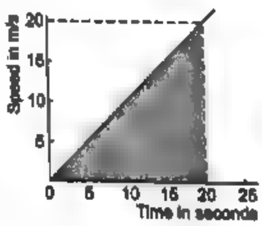


Fig. 2.23

Solution The distance covered in the time interval 0 to 20 s is equal to the area of the shaded triangle. It is

$$\frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times (20 \text{ s}) \times (20 \text{ m/s}) = 200 \text{ m}.$$

Velocity–Time Graph

Consider an object moving along a straight line. We choose and fix a particular direction along the line as the positive direction. If the object moves in the positive direction, its velocity is positive. In fact, it is the same as its speed. If the object moves in the negative direction, its velocity is negative. In this case, the velocity is the negative of its speed.

If a graph is plotted taking the velocity of an object moving along a straight line on the vertical axis and time on the horizontal axis, we get a velocity–time graph.

If the particle moves with a constant velocity v, the velocity–time graph will be a straight line parallel to the time-axis, as shown in Figure 2.24. The displacement s in time t is given by

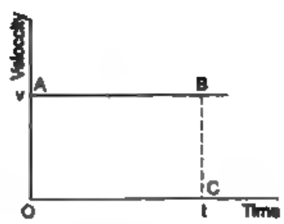


Fig. 2.24

$$s = vt = OA \cdot OC$$

= area of the rectangle $OABC$, which is the area under the graph.

So, the area under the velocity–time graph of an object gives its displacement. We have already seen that the area under the speed–time graph gives the distance covered.

Let us take an example. A ball is dropped from a height. We take the downward direction as positive. As the ball falls, its velocity increases. Table 2.6 shows the velocity of the ball at different instants. The velocity versus time graph is shown in Figure 2.25.

Table 2.6 Velocity of the falling ball at different instants

Time in s	Velocity in m/s
0	0
0.1	1
0.2	2
0.3	3
0.4	4
0.5	5
0.6	6

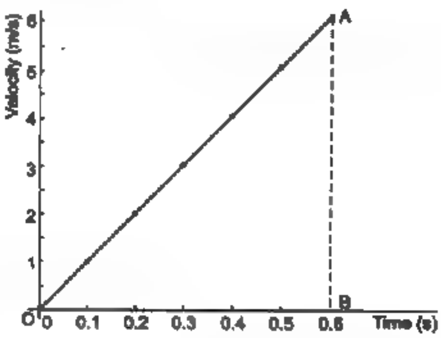


Fig. 2.25

We see that the plotted points fall on a straight line. What is the displacement of the ball in the time interval 0 to 0.6 s? It is equal to the area under the velocity–time graph from $t = 0$ to $t = 0.6$ s. This area is in the shape of a triangle. The area is

$$\frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times (OB) \times (AB)$$

$$= \frac{1}{2} \times (0.6 \text{ s}) \times (6 \text{ m/s}) = 1.8 \text{ m}.$$

The ball has fallen through 1.8 m in 0.6 s.

Next consider the situation shown in Figure 2.16. A ball placed on a smooth floor moves towards a wall, hits the wall at time $t = 7$ s (Table 2.5a) and returns (Table 2.5b). We take the direction towards the wall as positive and the direction away from the wall as negative. From time 0 to 7 seconds, the velocity is $+10$ cm/s, and from 7 to 14 seconds it is -10 cm/s. What will the velocity–time graph look like for the whole period 0 – 14 s?

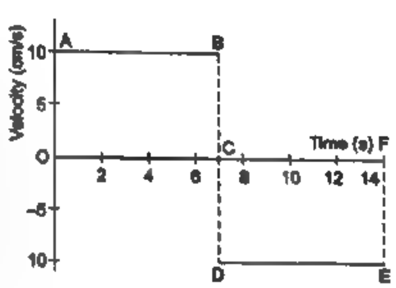


Fig. 2.26

Figure 2.26 shows the graph. The 'height' of the rectangle $OABC$ is positive and that of the rectangle $CDEF$ is negative.

The area under the curve for 0–7 s is

$$(OC)\times(OA)=(7\text{ s})\times\left(10\,\frac{\text{cm}}{\text{s}}\right)=70\text{ cm},$$

which is the displacement in the period 0 – 7 s.

The area under the curve for 7–14 is

$$(CF)\times(CD)=(7\text{ s})\times\left(-10\,\frac{\text{cm}}{\text{s}}\right)=-70\text{ cm},$$

which is the displacement in the period 7–14 s.

The total area under the curve for 0–14 s is

$$(70\text{ cm})+(-70\text{ cm})=0.$$

The displacement in the period 0–14 s is indeed zero, as the ball returns to its original position.

EXAMPLE 2.17 **What can you say about the nature of the motions of the particles for which the velocity–time graphs are given below?**

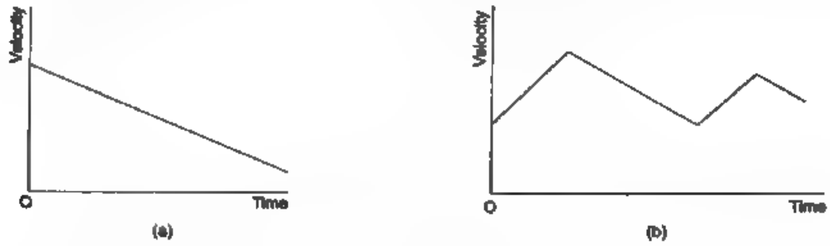


Fig. 2.27

Solution In case of Figure 2.27a, as time passes, the velocity decreases continuously. So, the particle is slowing down continuously. We see this type of motion when we throw a ball up. The ball slows down continuously on its way up.

In case of Figure 2.27b, the velocity increases and decreases alternately. As the velocity remains positive through out, the particle keeps moving in the same direction. You have this type of motion when a driver drives a car on a straight, busy road. He has to slow down (brake) and speed up (accelerate) alternately for a large part of the drive.

Acceleration from Velocity–Time Graph

Suppose a particle moves with a uniform acceleration of 2 m/s^2 along a straight line. This means that the velocity increases by 2 m/s in one second. Also, suppose its velocity at $t = 0$ is 10 m/s.

Let us plot the velocity–time graph for this situation. We first find the values of the velocity at certain instants. At $t = 0$, the velocity is 10 m/s. At $t = 1$ s it will become 10 m/s + 2 m/s = 12 m/s. At $t = 2$ s, it will become 14 m/s, and so on. These values are given in Table 2.7 and the velocity–time graph is shown in Figure 2.28.

Table 2.7

Time (s)	Velocity (m/s)
0	10
1	12
2	14
3	16
4	18
5	20

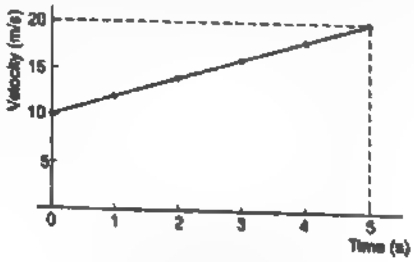


Fig. 2.28

We see that the graph is a straight line. When the acceleration is uniform, the velocity–time graph is a straight line. We will now show that the slope of the velocity–time graph gives the acceleration.

Suppose the velocity–time graph of a particle moving along a straight line is as shown in Figure 2.29. The graph is a straight line. At time t_1, the velocity is v_1, and at time t_2, it is v_2. These values are represented by the points A and B on the graph.

The acceleration of the object is

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{OE - OD}{OG - OF} = \frac{DE}{FG} = \frac{BC}{AC}.$$

As defined earlier, the ratio $\frac{BC}{AC}$ is called the slope of the line. Thus, we have the following.

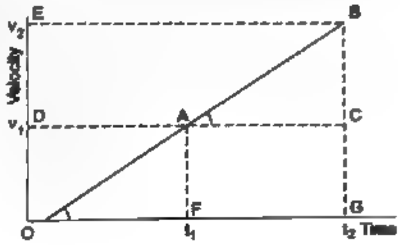


Fig. 2.29

> The slope of the velocity–time graph gives the acceleration for an object moving along a straight line.

EXAMPLE 2.10 **Figure 2.30 shows the velocity–time graphs for two objects, A and B, moving along the same direction. Which object has greater acceleration?**

Solution The slope of the velocity–time graph for B is greater than that for A. Thus, the acceleration of B is greater than that of A.

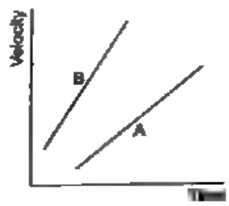


Fig. 2.30

<u>*Velocity–time graph for nonuniform acceleration*</u>

If the acceleration of an object moving along a straight line is not constant, the velocity–time graph is not a straight line. The graph in Figure 2.31 represents a motion in which acceleration increases with time.

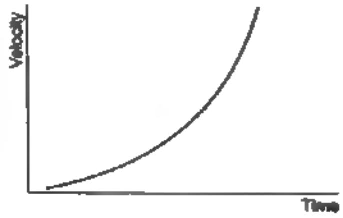


Fig. 2.31

EQUATIONS FOR MOTION WITH UNIFORM ACCELERATION

The motion of a particle moving along a straight line with a uniform acceleration can be described by three equations. These equations can be derived using a velocity–time graph.

Figure 2.32 shows the velocity–time graph of a particle moving along a straight line with a uniform acceleration a.

The points A and B on the graph correspond to the times 0 and t respectively.

The velocity of the particle at time $t = 0$ is u. In the graph, OA represents u.

The velocity of the particle at time t is v. In the graph, OD represents v.

Drop the perpendicular BE on the time-axis. Also, drop the perpendicular AC on BE.

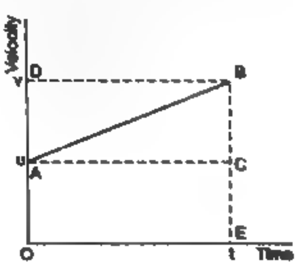


Fig. 2.32

Velocity–Time Equation

The slope of the velocity–time graph gives the acceleration of a particle moving in a straight line.

For the line AB, slope $= \dfrac{BC}{AC}$ (by definition)

or $$a = \frac{BC}{AC} = \frac{BE-CE}{OE} = \frac{OD-OA}{OE}$$

$$= \frac{v-u}{t}$$

or $$at = v - u$$

or $$\boxed{v = u + at}$$

Displacement–Time Equation

The area under the velocity–time graph is equal to the displacement. In the time interval 0–t, the displacement s is equal to the area $OABE$.

$$s = \text{area } OABE$$

$$= \text{area of the rectangle } OACE + \text{area of the triangle } ABC$$

$$= (OA)\cdot(OE) + \frac{1}{2}(AC)\cdot(BC)$$

$$= (OA)\cdot(OE) + \frac{1}{2}(OE)\cdot\left(\frac{BC}{AC}\cdot AC\right)$$

$$= (OA)\cdot(OE) + \frac{1}{2}(OE)\cdot\left(\frac{BC}{AC}\cdot OE\right)$$

$$= (OA)\cdot(OE) + \frac{1}{2}\left(\frac{BC}{AC}\right)(OE)^2.$$

Now, $OA = u$, $OE = t$ and $\dfrac{BC}{AC} = \text{slope} = a$. Putting these in the above equation,

$$\boxed{s = ut + \frac{1}{2}at^2} \qquad \ldots 2.4$$

Velocity–Displacement Equation

The area of a trapezium $= \dfrac{1}{2}$ (sum of the parallel sides) × (distance between them).

So, the area of the trapezium $OABE$ can be written as

$$s = \frac{1}{2}(OA + EB)\cdot(AC)$$

$$= \frac{1}{2}(OA + OD)\cdot\left(\frac{AC}{BC}\cdot BC\right)$$

$$= \frac{1}{2}(OA + OD)\cdot\left(\frac{BC - CE}{BC/AC}\right)$$

$$= \frac{1}{2}(OA + OD)\cdot\left(\frac{OD - OA}{BC/AC}\right)$$

$$= \frac{1}{2}\left(\frac{OD^2 - OA^2}{BC/AC}\right) = \frac{1}{2}\left(\frac{v^2 - u^2}{a}\right)$$

or $$2as = v^2 - u^2$$

or $$\boxed{v^2 = u^2 + 2as} \qquad \text{...2.5}$$

Average Velocity for Uniformly Accelerated Motion

Consider a particle moving along a straight line with a uniform (constant) acceleration a.

The average velocity of the particle during time $0-t$ is

$$v_{av} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{s}{t} = \frac{ut + \frac{1}{2}at^2}{t}$$

$$= u + \left(\frac{at}{2}\right)$$

$$= u + \left(\frac{v-u}{2}\right) \qquad [\quad v = u + at]$$

or $$\boxed{v_{av} = \frac{(u+v)}{2}} \qquad \text{...2.6}$$

So, for motion along a straight line with uniform acceleration, the average velocity is just the average of the initial velocity and the final velocity.

EXAMPLE 2.19 **A particle moving with an initial velocity of 5.0 m/s is subjected to a uniform acceleration of $-2.5\ \text{m/s}^2$. Find the displacement in the next 4.0 s.**

Solution The displacement is

$$s = ut + \frac{1}{2}at^2$$

$$= \left(5.0\frac{\text{m}}{\text{s}}\right) \times (4.0\ \text{s}) + \frac{1}{2}\left(-2.5\frac{\text{m}}{\text{s}^2}\right) \times (4.0\ \text{s})^2 = 20\ \text{m} - 20\ \text{m} = 0.$$

So, after 4.0 s, the particle will be back at its initial position. Note that the distance traversed is not zero, as the particle moves in the forward direction and then comes back to the initial position.

Displacement–Time Graph for Uniformly Accelerated Motion

The displacement of a particle moving along a straight line with uniform acceleration a is given by

$$s = ut + \frac{1}{2}at^2.$$

If the particle starts from rest, $a = 0$, and

$$s = \frac{1}{2}at^2.$$

If we plot the displacement–time graph for such a particle, the graph will be a curved line as shown in Figure 2.33. Such a curve is known as a parabola.

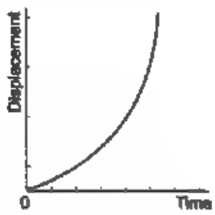
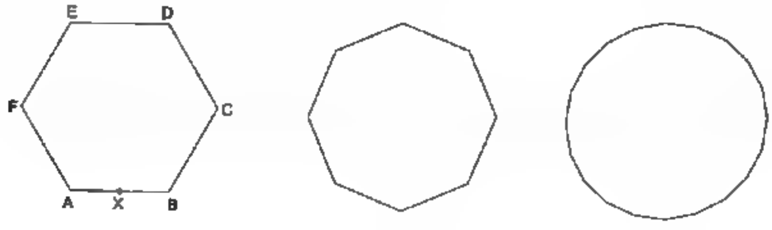


Fig. 2.33

CIRCULAR MOTION

Suppose an object moves along a circular path in such a way that it covers equal distances in equal time intervals. The speed of the object is, therefore, constant. Is the velocity of the object also constant? To answer this question, we have to find whether or not the object has changed its direction of motion while moving.



(a) (b) (c)

Fig. 2.34

Let us first consider a hexagonal path *ABCDEFA* (Figure 2.34a) in which all the sides are equal. Suppose an object moves with a uniform speed along this path. Consider a complete round, starting from the point *X* and ending there. To stay on the path, the object has to change its direction of motion quickly at the corners *B, C, D, E, F* and *A*. So, in one complete round, the object changes its direction six times. Instead, if it moves along an octagonal path (Figure 2.34b), it has to change its direction 8 times during a complete round. Suppose we continue to increase the number of sides and at the same time decrease the length of the sides so that the total length of the path is fixed. The object has to change its direction at every corner, with the corners occurring more frequently. Figure 2.34c shows a polygonal path of 20 sides. Its shape is quite close to that of a circle. The object changes its direction at every corner—20 times in a complete round. If we make the number of sides very large and the length of each side very small, the polygon becomes almost a circle. It will now have a very large number of corners, which occur at almost every point of the path. If the object moves along such a path, i.e., along a circle, it has to change its direction continuously, at every instant.

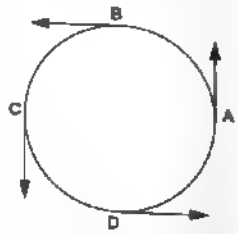


Fig. 2.35

The direction of motion of an object moving along a circular path can be found in a simple manner. Draw a tangent to the circle from the object's position on the circle. The direction of motion at that instant is along this tangent. The arrows in Figure 2.35 show the directions of motion of the object at various positions. We see that a particle moving along a circular path changes its direction continuously. Its velocity is, therefore, not constant even if its speed is constant. Circular motion is, therefore, an example of accelerated motion.

If r be the radius of the circle and v be the speed of a particle moving along it, how much time (t) does the particle take to complete one revolution? The length covered is $2\pi r$. Thus,

$$v = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi r}{v}$$

SOLVED PROBLEMS

EXAMPLE 1 **A particle moves through a distance of 3 m due east and then 4 m due north. (a) How much is the net distance traversed? (b) What is the magnitude of the net displacement?**

Solution The situation is shown in Figure 2.W1. The particle starts from O. It moves through a distance of 3 m due east to reach A and then through a distance of 4 m due north to reach B.

(a) The total distance moved is 3 m + 4 m = 7 m.

(b) The magnitude of the net displacement is OB. In the right-angled triangle OAB,

$$OB^2 = OA^2 + AB^2$$
$$= (3\ \text{m})^2 + (4\ \text{m})^2$$
$$= 9\ \text{m}^2 + 16\ \text{m}^2 = 25\ \text{m}^2$$

or $$OB = 5\ \text{m}.$$

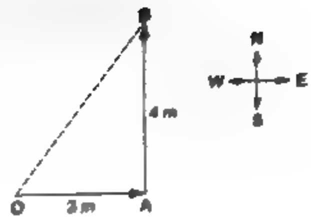


Fig. 2.W1

EXAMPLE 2 **A car covers 30 km in 30 minutes and the next 30 km in 40 minutes. Calculate the average speed for the entire journey.**

Solution As given, the total time taken is 30 min + 40 min = 70 min, and the total distance traversed is 30 km + 30 km = 60 km. The average speed is

$$v_{av} = \frac{60\ \text{km}}{70\ \text{min}} = \frac{60\ \text{km}}{(70/60)\ \text{h}} = \frac{3600}{70}\ \text{km/h} \approx 51.4\ \text{km/h}.$$

EXAMPLE 3 **A car covers 30 km at a uniform speed of 30 km/h. What should be its speed for the next 90 km if the average speed for the entire journey is 60 km/h?**

Solution The total distance = 30 km + 90 km = 120 km.

The average speed for the entire journey = 60 km/h.

Using $s = vt$,

$$t = \frac{s}{v} = \frac{120\ \text{km}}{60\ \text{km/h}} = 2\ \text{h}.$$

Thus, it takes 2 hours to complete the journey. The first 30 km is covered at a speed of 30 km/h. Suppose the time taken to cover the first 30 km is t_1. Using $s = vt$,

$$t_1 = \frac{30\ \text{km}}{30\ \text{km/h}} = 1\ \text{h}.$$

Thus, the remaining 90 km must be covered in (2 h − 1 h) = 1 h. The speed during this 90 km should be

$$v = \frac{s}{t} = \frac{90\ \text{km}}{1\ \text{h}} = 90\ \text{km/h}.$$

EXAMPLE 4 **A boy runs for 10 min at a uniform speed of 9 km/h. At what speed should he run for the next 20 min so that the average speed comes to 12 km/h?**

Solution Total time = 10 min + 20 min = 30 min.

The average speed is 12 km/h.

Using $s = vt$, the total distance covered in 30 min is 12 km/h × 30 min = $12\ \frac{\text{km}}{\text{h}} \times \frac{1}{2}\ \text{h} = 6\ \text{km}$.

The distance covered in the first 10 min is

$$9\ \text{km/h} \times 10\ \text{min} = 9\ \frac{\text{km}}{\text{h}} \times \frac{1}{6}\ \text{h} = 1.5\ \text{km}.$$

Thus, he has to cover 6 km − 1.5 km = 4.5 km in the next 20 min. The speed required is

$$\frac{4.5\ \text{km}}{20\ \text{min}} = \frac{4.5\ \text{km}}{(20/60)\ \text{h}} = 13.5\ \text{km/h}.$$

EXAMPLE 5 **A particle was at rest from 9.00 a.m. to 9.30 a.m. It moved at a uniform speed of 10 km/h from 9.30 a.m. to 10.00 a.m. Find the average speed between (a) 9.00 a.m. and 10.00 a.m., (b) 9.15 a.m. and 10.00 a.m.**

Solution (a) The distance moved by the particle between 9.30 a.m. and 10.00 a.m. is

$$s = vt = 10\ \frac{\text{km}}{\text{h}} \times \frac{1}{2}\ \text{h} = 5\ \text{km}.$$

This is also the distance moved between 9.00 a.m. and 10.00 a.m. Thus, the average speed during this interval is

$$v_{av} = \frac{s}{t} = \frac{5\text{ km}}{1\text{ h}} = 5\text{ km/h}.$$

(b) The distance moved between 9.30 a.m. and 10.00 a.m. is 5 km. This is also the distance moved in the interval 9.15 a.m. to 10.00 a.m. The average speed during this interval is

$$v_{av} = \frac{s}{t} = \frac{5\text{ km}}{45\text{ min}} = \frac{5\text{ km}}{(45/60)\text{ h}} = \frac{5\times 60}{45}\text{ km/h} \approx 6.67\text{ km/h}.$$

EXAMPLE 6 **An insect moves along a circular path of radius 10 cm with a constant speed. If it takes 1 minute to move from a point on the path to the diametrically opposite point, find (a) the distance covered, (b) the speed, (c) the displacement, and (d) the average velocity.**

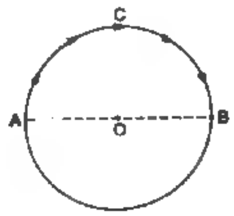


Fig. 2.W2

Solution Suppose the insect was at A initially, and it moved along ACB to reach the diametrically opposite point B in 1 minute.

(a) The distance moved in 1 minute $= \pi r = 3.14 \times 10\text{ cm} = 31.4\text{ cm}$.

(b) The speed is $\frac{31.4\text{ cm}}{1\text{ min}} = 31.4\text{ cm/min}$.

(c) The displacement is $AB = 2r = 20\text{ cm}$ in the direction A to B.

(d) The average velocity is

$$v_{av} = \frac{\text{displacement}}{\text{time}} = \frac{20\text{ cm}}{1\text{ min}} = 20\text{ cm/min in the direction } A \text{ to } B.$$

EXAMPLE 7 **What is wrong with the following graphs?**

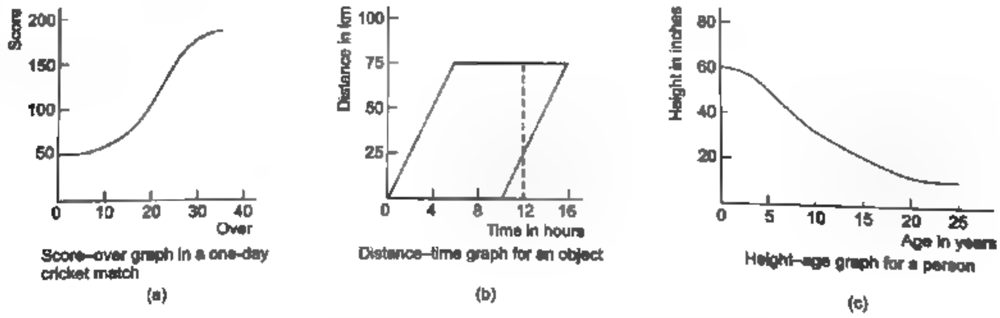


Fig. 2.W3

Solution (a) 50 runs are shown at the end of the 0th over, i.e., at the beginning. This is not possible in a one-day cricket match under the present rules.

(b) If we draw a perpendicular on the time-axis at the point corresponding to 12 hours, it cuts the graph at two points. One corresponds to 25 km and the other corresponds to 75 km. Thus, according to the graph, the distance travelled in 12 hours is 25 km as well as 75 km, which is not possible.

(c) According to the graph, the height of a person gradually decreases as his age increases. Such a thing does not happen.

EXAMPLE 8 The distance–time table for a car is given. Assuming that the car moves with uniform speed between the indicated times, answer the following questions.

Time	Distance in km
10.05 a.m.	0
10.25 a.m.	5
10.40 a.m.	12
10.50 a.m.	22
11.00 a.m.	26
11.10 a.m.	28
11.25 a.m.	38
11.40 a.m.	42

(a) Plot the graph of the distance travelled with time.
(b) During which period was the car travelling at the greatest speed?
(c) During which period was the car moving with the least speed?
(d) What is the average speed of the car between 10.05 a.m. and 11.00 a.m.?
(e) What is the average speed of the car for the entire journey?

Solution (a)

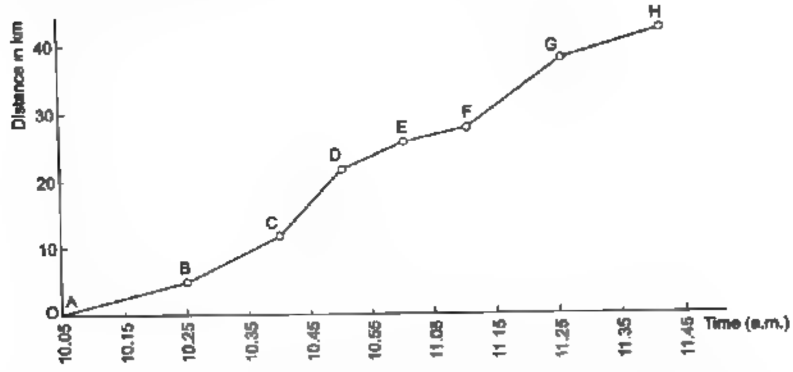


Fig. 2.W4

The graph is shown in Figure 2.W4. The consecutive points are joined by straight lines. This is because we have assumed that the car moves with uniform speed in each interval.

(b) The greatest inclination (slope) with the time-axis occurs in the part *CD* of the graph. Thus, the speed is the greatest in this part, i.e., between 10.40 a.m. and 10.50 a.m.

(c) The least inclination with the time-axis occurs in the part *EF*. Thus, the speed is the least in this part, i.e., between 11.00 a.m. and 11.10 a.m.

(d) The distance travelled between 10.05 a.m and 11.00 a.m is 26 km. The time interval is 55 min. The average speed is

$$v = \frac{26\ \text{km}}{55\ \text{min}} = \frac{26\ \text{km}}{(55/60)\ \text{h}} = \frac{26 \times 60}{55}\ \text{km/h} \approx 28.4\ \text{km/h}.$$

(e) The total distance travelled is 42 km and the total time taken is 1 h 35 min. The average speed is

$$v = \frac{42\ \text{km}}{1\ \text{h}\ 35\ \text{min}} = \frac{42\ \text{km}}{\left(1+\frac{35}{60}\right)\text{h}} = \frac{42 \times 60}{95}\ \text{km/h} \approx 26.5\ \text{km/h}.$$

EXAMPLE 9 A train is moving at a speed of 40 km/h at 10.00 a.m. and at 50 km/h at 10.02 a.m. Assuming that the train moves along a straight track and the acceleration is constant, find the value of the acceleration.

Solution The acceleration is

$$a = \frac{v-u}{t} = \frac{50\ \text{km/h} - 40\ \text{km/h}}{2\ \text{min}}$$

$$= \frac{10\ \text{km/h}}{(2/60)\ \text{h}} = \frac{10 \times 60}{2}\ \text{km/h}^2 = 300\ \text{km/h}^2.$$

EXAMPLE 10 **A particle with a velocity of 2 m/s at $t = 0$ moves along a straight line with a constant acceleration of 0.2 m/s ^{2}. Find the displacement of the particle in 10 s.**

Solution

$$s = ut + \frac{1}{2}at^2 = (2\ \text{m/s})(10\ \text{s}) + \frac{1}{2}(0.2\ \text{m/s}^2)(10\ \text{s})^2$$
$$= 20\ \text{m} + 10\ \text{m} = 30\ \text{m}.$$

EXAMPLE 11 **A particle is pushed along a horizontal surface in such a way that it starts with a velocity of 12 m/s. Its velocity decreases at a rate of 0.5 m/s ^{2}. (a) Find the time it will take to come to rest. (b) Find the distance covered by it before coming to rest.**

Solution

(a) Initial velocity $u = 12$ m/s.
Acceleration $a = -0.5\ \text{m/s}^2$ (as the velocity is decreasing).
Final velocity $u = 0$ (as it comes to rest).
We have $v = u + at$

or $0 = (12\ \text{m/s}) + (-0.5\ \text{m/s}^2)t$

or $(0.5\ \text{m/s}^2)t = 12\ \text{m/s}$

or $t = \dfrac{12\ \text{m/s}}{0.5\ \text{m/s}^2} = \dfrac{12}{0.5}\dfrac{\text{m}}{\text{s}} \times \dfrac{\text{s}^2}{\text{m}} = 24\ \text{s}.$

So, the particle takes 24 s to stop.

(b) We have $v^2 = u^2 + 2as$

or $0 = (12\ \text{m/s})^2 + 2(-0.5\ \text{m/s}^2)s$

or $(1\ \text{m/s}^2)s = 144\ \text{m}^2/\text{s}^2$

or $s = \dfrac{144\ \text{m}^2/\text{s}^2}{1\ \text{m/s}^2} = 144\ \text{m}.$

So, the particle covers 144 m before stopping.

EXAMPLE 12 **A train accelerates from 20 km/h to 80 km/h in 4 minutes. How much distance does it cover in this period? Assume that the tracks are straight.**

Solution We will first find the acceleration and then the distance.
At $t = 0$, the velocity is $u = 20$ km/h.
At $t = 4\ \text{min} = \frac{1}{15}$ h, the velocity is $v = 80$ km/h.
Using $v = u + at$,

$$a = \frac{v-u}{t} = \frac{80\ \text{km/h} - 20\ \text{km/h}}{(1/15)\ \text{h}} = 60\ \text{km/h}^2 \times 15 = 900\ \text{km/h}^2.$$

The distance covered is

$$x = ut + \frac{1}{2}at^2 = (20\ \text{km/h}) \times \left(\frac{1}{15}\text{h}\right) + \frac{1}{2}(900\ \text{km/h}^2) \times \left(\frac{1}{15}\text{h}\right)^2 = \frac{20}{15}\ \text{km} + 2\ \text{km} = \frac{10}{3}\ \text{km}.$$

EXAMPLE 13 **A car moving along a straight line at a speed of 54 km/h stops in 5 seconds after the brakes are applied. (a) Find the acceleration, assuming it to be constant. (b) Plot the graph of speed versus time. (c) Using the graph, find the distance covered by the car after the brakes are applied.**

Solution

(a) The initial velocity is

$$u = 54\ \text{km/h} = \frac{54\ \text{km}}{1\ \text{h}} = \frac{54 \times 1000\ \text{m}}{3600\ \text{s}} = 15\ \text{m/s}.$$

The velocity v at $t = 5$ s is zero.
The acceleration is

$$a = \frac{v-u}{t} = \frac{0 - 15\ \text{m/s}}{5\ \text{s}} = -3\ \text{m/s}^2.$$

Since the sign of the acceleration is opposite to that of the velocity, the car is decelerating.

(b) At $t = 0$, the speed is 15 m/s, and at $t = 5$ s, it is 0. Thus, we get two points A and B on the graph (Figure 2.W5). As the acceleration is constant and the car moves in a fixed direction, the speed–time graph should be a straight line. Therefore, we join the points A and B by a straight line to get the required graph.

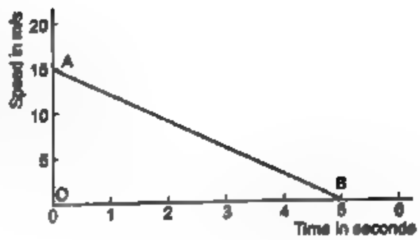


Fig. 2.W5

(c) The distance covered between 0 and 5 s is equal to the area under the graph, i.e., the area of the triangle OAB. It is

$$\frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2} \times (15 \text{ m/s}) \times (5 \text{ s}) = 37.5 \text{ m}.$$

EXAMPLE 14 **Figure 2.W6 shows the speed–time graph of a particle. Find the distance travelled in the time interval 0 to 40 s.**

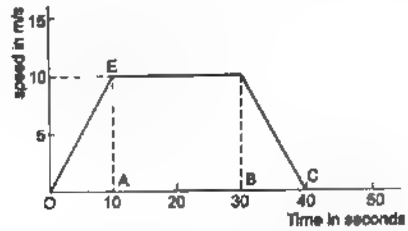


Fig. 2.W6

Solution The distance travelled is equal to the area under the graph From Figure 2.W6, this area is equal to the area of the triangle OAE + the area of the rectangle $ABDE$ + the area of the triangle BCD.

The area of the triangle OAE is

$$A_1 = \frac{1}{2} \times OA \times AE = \frac{1}{2} \times (10 \text{ s}) \times (10 \text{ m/s}) = 50 \text{ m}.$$

The area of the rectangle $ABDE$ is

$$A_2 = AE \times AB = (10 \text{ m/s}) \times (20 \text{ s}) = 200 \text{ m}.$$

The area of the triangle BCD is

$$A_3 = \frac{1}{2} \times BC \times BD = \frac{1}{2} \times (10 \text{ s}) \times (10 \text{ m/s}) = 50 \text{ m}.$$

The total area is $A_1 + A_2 + A_3 = 50 \text{ m} + 200 \text{ m} + 50 \text{ m} = 300 \text{ m}$.

The total distance covered is 300 m.

EXAMPLE 15 **The velocity–time graph of a particle moving along a straight line is shown in Figure 2.W7.**

(a) Is the motion uniform?

(b) Is the acceleration uniform?

(c) Does the particle change its direction of motion?

(d) Find the distances covered from 0 to 4 s and from 4 to 6 s.

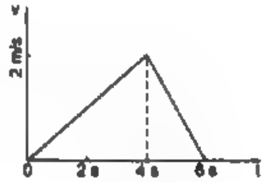


Fig. 2.W7

Solution (a) The velocity is changing with time. So, the motion is not uniform.

(b) The acceleration is given by the slope of the velocity–time graph The slopes are different before and after $t = 4$ s. So, the acceleration is not uniform for the entire time shown. It is uniform between 0 and 4 s and also between 4 and 6 s as the slope does not change in these periods.

(c) The velocity always remains positive. It means that the particle keeps moving in the positive direction. In other words, it does not change direction.

(d) The displacement during the period 0–4 s is equal to the area under the velocity–time graph for this period. This area is in the shape of a triangle.

$$\text{Area of the triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 4\ \text{s} \times (2\ \text{m/s}) = 4\ \text{m}.$$

As the particle moves in the same direction, this is also the distance moved.

For the period 4–6 s, the area is

$$\frac{1}{2}(2\text{s}) \times 2\left(\frac{\text{m}}{\text{s}}\right) = 2\ \text{m}.$$

So, the particle moves 2 m in this period.

• POINTS TO REMEMBER •

- *Rest and Motion*

  If the position of an object does not change as time passes, it is said to be at rest. If it changes as time passes, the object is said to be in motion.

- *Describing position*

  The position of a particle is described by giving

  (a) its distance from a fixed point called the origin, and

  (b) its direction as seen from the origin.

  The position of a particle moving along a straight line is described by a quantity x. The numerical value of x gives the distance of the particle from the origin, and its sign indicates whether the particle is in the positive direction or in the negative direction as seen from the origin.

- *Displacement*

  The change in the position of a particle during a time interval is called its displacement in that time interval. The displacement tells us two things:

  (a) how far the final position is from the initial position, and

  (b) in which direction the final position is as seen from the initial position.

- *Average speed*

  $$\text{average speed} = \frac{\text{distance traversed}}{\text{time interval}}$$

- *Speed*

  The speed of an object is equal to the distance traversed by it in a short time interval divided by the time interval.

- *Uniform speed*

  If an object covers equal distances in equal time intervals, however small the intervals be, it is said to move with a uniform speed.

- *Velocity*

  The velocity of an object is the displacement of the object in a short time interval divided by the time interval. Velocity changes if either speed or the direction of motion changes.

- *Acceleration*

  (a) The acceleration of an object is equal to the change in its velocity per unit time.

  (b) If the velocity of an object changes by equal amounts in equal time intervals, the object is said to have a uniform acceleration.

- *Distance–time graph*

  (a) The distance–time graph of an object moving with a uniform speed is a straight line. Conversely, if the distance–time graph of an object is a straight line, the object is moving with a uniform speed.

  (b) The slope of the distance–time graph of an object equals its speed.

  (c) If an object moves with nonuniform speed, its distance–time graph is not a straight line.

- *Displacement–time graph*

  (a) The displacement–time graph of an object moving with a uniform velocity is a straight line.

  (b) The slope of the displacement–time graph of an object equals its velocity.

- *Speed–time graph*

  (a) If an object moves with a constant speed, its speed–time graph is a straight line parallel to the time-axis.

  (b) The area under the speed–time graph gives the distance traversed by the object in the corresponding time interval.

- *Velocity–time graph*
  (a) If an object moves with a constant acceleration in a straight line, its velocity–time graph is a straight line.
  (b) The slope of the velocity–time graph gives the acceleration of the object.
  (c) The area under a velocity–time graph gives the displacement of the object.
- *Circular motion*
  A particle moving in a circular path changes its direction continuously, and hence, is accelerated.
- *Mathematical equations*

$s = vt$	s = distance, v = speed (assumed constant), t = time
$v = u + at$	u = velocity at $t = 0$, v = velocity at time t, a = acceleration (assumed constant)
$s = ut + \frac{1}{2}at^2$	s = displacement during time 0 to t, u = velocity at $t = 0$, a = acceleration (assumed constant)
$v^2 = u^2 + 2as$	symbols have the same meaning as in the above equations

EXERCISES

A. *Objective Questions*

I. *Pick the correct option.*

1. A particle is travelling with a constant speed. This means
   (a) its position remains constant as time passes
   (b) it covers equal distances in equal time intervals
   (c) its acceleration is zero
   (d) it does not change its direction of motion
2. A particle moves with a uniform velocity.
   (a) The particle must be at rest.
   (b) The particle moves along a curved path.
   (c) The particle moves along a circle.
   (d) The particle moves along a straight line.
3. If a particle covers equal distances in equal time intervals, it is said to
   (a) be at rest
   (b) move with a uniform speed
   (c) move with a uniform velocity
   (d) move with a uniform acceleration
4. A quantity has a value of –6.0 m/s. It may be the
   (a) speed of a particle
   (b) velocity of a particle
   (c) acceleration of a particle
   (d) position of a particle
5. The area under a graph between two quantities is given in the unit m/s. The quantities are
   (a) speed and time
   (b) distance and time
   (c) acceleration and time
   (d) velocity and time
6. The area under a speed–time graph is represented by the unit
   (a) m (b) m^2 (c) m^3 (d) m^{-1}
7. The velocity–time graph of a particle is not a straight line. Its acceleration is
   (a) zero (b) constant
   (c) negative (d) variable
8. If a particle moves with a constant speed, the distance–time graph is a
   (a) straight line (b) circle
   (c) stairlike line (d) polygon
9. The distance–time graph of an object moving in a fixed direction is shown in Figure 2.E1.

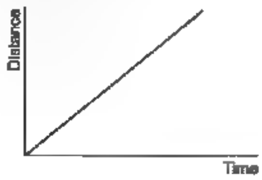


Fig. 2.E1

   The object
   (a) is at rest
   (b) moves with a constant velocity
   (c) moves with a variable velocity
   (d) moves with a constant acceleration
10. The distance–time graph of an object is shown in Figure 2.E2.

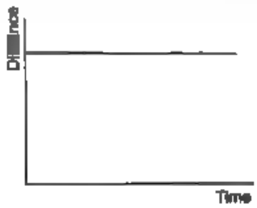


Fig. 2.E2

   The object
   (a) is at rest
   (b) moves with a constant speed
   (c) moves with a constant velocity
   (d) moves with a constant acceleration

11. The speed–time graph of an object moving in a fixed direction is shown in Figure 2.E3.

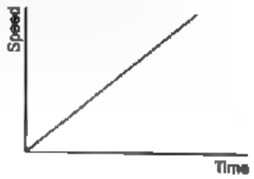


Fig. 2.E3

The object
(a) is at rest
(b) moves with a constant speed
(c) moves with a constant velocity
(d) moves with a constant acceleration

12. The speed–time graph of an object moving in a fixed direction is shown in Figure 2.E4.

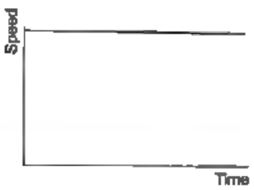


Fig. 2.E4

The object
(a) is at rest
(b) moves with fluctuating speed
(c) moves with a constant speed
(d) moves with a nonzero acceleration

13. In circular motion the
(a) direction of motion is fixed
(b) direction of motion changes continuously
(c) acceleration is zero
(d) velocity is constant

II. *Mark the statements true (T) or false (F).*

1. If A moves with respect to B then B moves with respect to A.
2. Scalar quantities can be added according to the rules of arithmetic.
3. The magnitude of the displacement of a particle can be greater than the distance traversed.
4. The magnitude of the displacement of a particle can be equal to the distance traversed.
5. Vector quantities can be added according to the rules of arithmetic.
6. The displacement of a particle in a 10-minute interval is zero. Its velocity at every instant in this interval must be zero.
7. A particle is known to be at rest at time $t = 0$. Its acceleration at $t = 0$ must be zero.
8. For a particle moving with a constant velocity, the distance–time graph is a straight line.
9. For a particle moving with a constant acceleration along a straight line, the velocity–time graph is a straight line

III. *Fill in the blanks.*

1. A vector quantity has magnitude as well as ....
2. Distance is a ..... quantity as it has no direction.
3. Displacement is a ... quantity.
4. km/h^2 is a unit of ....
5. The slope of a distance–time graph gives ....
6. The speed–time graph for a particle moving at a constant speed is a straight line ...... to the time-axis.
7. When an object moves in a fixed direction with a uniform acceleration, the speed–time graph is a ....
8. The area under the speed–time graph gives the ....
9. The area under the velocity–time graph gives the ....

B. *Very-Short-Answer Questions*

Answer the following in one word or maximum one sentence.

1. You are walking towards India Gate. Is India Gate at rest with respect to you or is it moving with respect to you?
2. Which of the following are scalar quantities?
(a) Mass (b) Displacement
(c) Speed (d) Velocity
3. Here are certain positions of a particle which can move in a horizontal plane. Two of them denote identical positions. Identify these positions.
(a) 5 m, 30° north of east
(b) 5 m, 30° east of north
(c) 5 m, 60° south of west
(d) 5 m, 60° east of north
4. What is the displacement of a satellite when it makes a complete round along its circular path?
5. A scooter moves 45 km on one litre of petrol. In a journey, the scooter used up one litre of petrol. Is it necessary that the displacement of the scooter in the journey is 45 km? Is it possible that the displacement is 45 km?
6. Can the distance travelled by an object be smaller than the magnitude of its displacement?
7. In what condition is the distance covered equal to the magnitude of the displacement of a particle?
8. A particle is moving with a uniform speed. Is it necessary that it is moving along a straight line?
9. A particle is moving with a uniform velocity. Is it necessary that it is moving along a straight line?
10. Which of the quantities—speed, velocity and acceleration—have the same SI unit?
11. Consider, three quantities—time, velocity and acceleration. The product of the units of two of these quantities gives the unit of the third. Which is the third quantity?

12. Can the equation $v = u + at$ be used for a particle moving with nonuniform acceleration?
13. A particle is moving with a uniform velocity. What is its acceleration?
14. Figure 2.E5 shows speed–time graphs for four cases. In which case is the speed constant? In which case is the speed decreasing? In which case is the speed increasing? What happens in the fourth case?

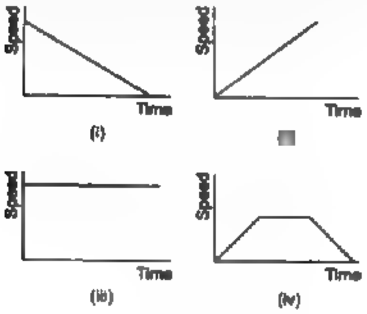


Fig. 2.E5

C. *Short-Answer Questions*

Answer the following in about 30–40 words each.

1. Give an example in which the distance traversed by a particle is larger than the magnitude of its displacement in the same time.
2. Define average speed
3. Define uniform speed.
4. Define uniform velocity.
5. A quantity is measured to be –30 m/s. Is it speed or velocity? Give a reason for your answer
6. What is the difference between speed and velocity?
7. Can the speed of a particle be negative? Can the velocity of a particle be negative? Give arguments to support your answers.
8. Define uniform acceleration.
9. Distinguish between distance and displacement for (a) an object moving along a straight line, and (b) an object moving in a plane.
10. Give an example where the speed of an object first increases, then remains constant for some time, and then decreases
11. How can you get the speed of an object from its distance–time graph?
12. How can you get the distance travelled by an object from its speed–time graph?
13. A girl bends to touch her toes. Is the motion of her head an example of uniform motion or accelerated motion? Explain your answer

D. *Long-Answer Questions*

Answer the following in not more than 70 words.

1. When do we say that a body is at rest and when do we say that it is moving?
2. Give two examples to explain that motion is relative.
3. How is the position of a particle moving along a straight line described by a number? How is the direction of motion specified by the number describing position?
4. What information about the initial and final positions of a particle do we get if the displacement is given?
5. How do you get the displacement of a particle in a time interval t_1 to t_2 from its velocity–time graph?
6. The distance–time graph for a particle is a straight line. Show that its speed is constant.
7. What are the steps involved in drawing a graph between two quantities?
8. What advantages does a graph have over a table having the same information in the form of numbers?
9. Show that the area under the speed–time graph for a particle moving at a constant speed gives the distance covered by the particle.

E. *Numerical Problems*

1. A car moves 100 m due east and then 25 m due west. (a) What is the distance covered by the car? (b) What is its displacement?
2. A person walks along the sides of a square field. Each side is 100 m long. What is the maximum magnitude of displacement of the person in any time interval?
3. In the hare–tortoise race, the hare ran for 2 min at a speed of 7.5 km/h, slept for 56 min and again ran for 2 min at a speed of 7.5 km/h. Find the average speed of the hare in the race.
4. A bus takes 8 hours to cover a distance of 320 km. What is the average speed of the bus?
5. The maximum speed of a train is 80 km/h. It takes 10 hours to cover a distance of 400 km. Find the ratio of its maximum speed to its average speed.
6. An object moves through 10 m in 2 minutes and next 10 m in 3 minutes. Calculate its average speed.
7. A car moves through 20 km at a speed of 40 km/h, and the next 20 km at a speed of 60 km/h. Calculate its average speed.
8. A boy leaves his house at 9.30 a.m. for his school. The school is 2 km away and classes start at 10.00 a.m. If he walks at a speed of 3 km/h for the first kilometre, at what speed should he walk the second kilometre to reach just in time?
9. A bus moves at a uniform speed v_1 for some time and then with a uniform speed v_2. The distance–time table is given below. Plot the corresponding distance–time graph and answer the following questions.

Time (min)	Distance (km)
0	0
20	20
40	40
60	65
80	95
100	125
120	155

(a) Find the values of v_1 and v_2.
(b) When did the bus change its speed?
(c) What is the distance covered in the first hour?
(d) What is the distance covered in the second hour?
(e) What is the average speed for the complete journey?

10. A bicycle increases its velocity from 10 km/h to 15 km/h in 6 seconds. Calculate its acceleration.
11. An object moves along a straight line with an acceleration of 2 m/s^2. If its initial speed is 10 m/s, what will be its speed 5 s later?
12. An object dropped from a cliff falls with a constant acceleration of 10 m/s^2. Find its speed 2 s after it was dropped.
13. A bullet hits a wall with a velocity of 20 m/s and penetrates it up to a distance of 5 cm. Find the deceleration of the bullet in the wall.
14. A train starts from a station and moves with a constant acceleration for 2 minutes. If it covers a distance of 400 m in this period, find the acceleration.
15. A ship moving with a constant acceleration of 36 km/h^2 in a fixed direction speeds up from 12 km/h to 18 km/h. Find the distance traversed by the ship in this period.
16. A particle starts from a point with a velocity of +6.0 m/s and moves with an acceleration of −2.0 m/s^2. Show that after 6 s the particle will be at the starting point.
17. A bicycle moves with a constant velocity of 5 km/h for 10 minutes and then decelerates at the rate 1 km/h^2, till it stops. Find the total distance covered by the bicycle.
18. An object is moving along a straight line with a uniform speed of 10 m/s. Plot a graph showing distance versus time from $t = 0$ to $t = 10$ s.
19. A particle moves along a straight line with a uniform velocity of 5.0 m/s. Plot a distance–time graph for the period $t = 0$ to $t = 5$ s.
20. The driver of a car travelling at 36 km/h applies the brakes to decelerate uniformly. The car stops in 10 s. Plot the speed–time graph for this period. Find the distance travelled by the car during this period by calculating the area under the graph.
21. A particle moves along a straight line with a constant acceleration $a = +0.5$ m/s^2. At $t = 0$ it is at $x = 0$, and its velocity is $v = 0$. Plot the velocity–time and position–time graphs for the period $t = 0$ to $t = 5$ s.
22. Figure 2.E6 shows the speed–time graph of a bus.

Fig. 2.E6

(a) In which period is the bus accelerating?
(b) In which period is the bus decelerating?
(c) What is the distance covered during its acceleration?
(d) What is the distance covered during its deceleration?
(e) What is the average speed in the entire journey?

23. The velocity–time graph of a particle moving along a straight line is given in Figure 2.E7. (a) Is the particle moving in the positive direction at $t = 0$? (b) Does the particle ever come to rest? If so, when? (c) Does the particle turn around? If so, when?

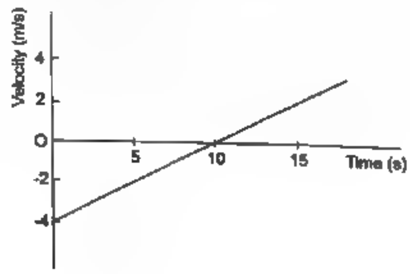


Fig. 2.E7

• ANSWERS •

A. *Objective Questions*

I. 1. (b) 2. (d) 3. (b) 4. (b) 5. (c)
6. (a) 7. (d) 8. (a) 9. (b) 10. (a)
11. (d) 12. (c) 13. (b)

II. 1. T 2. T 3. F 4. T 5. F
6. F 7. F 8. T 9. T

E. *Numerical Problems*

1. (a) 125 m (b) 75 m due east 2. $100\sqrt{2}$ m
3. 0.50 km/h 4. 40 km/h 5. 2 6. 4 m/min
7. 48 km/h 8. 6 km/h
9. (a) 60 km/h, 90 km/h (b) At 50 min
(c) 65 km (d) 90 km (e) 77.5 km/h
10. 3,000 km/h^2 11. 20 m/s 12. 20 m/s
13. 4,000 m/s^2 14. $\frac{1}{18}$ m/s^2 15. 2.5 km
17. $\frac{40}{3}$ km 20. 50 m
22. (a) 0–10 min (b) 10 min–20 min
(c) $\frac{10}{3}$ km (d) $\frac{10}{3}$ km (e) 20 km/h
23. (a) No (b) Yes, at $t = 10$ s (c) Yes, at $t = 10$ s

• POSTSCRIPT •

- *Does the earth move?*

  In the days of Aristotle it was believed that the earth remains at rest and the sun moves round it. Copernicus initiated the idea that the sun is at rest and the earth moves round it. Galileo was a firm believer of this theory and worked hard to let others know about it. He was warned by the Church not to spread this idea. As Galileo kept on doing so, his book *Dialogues of Galileo Galilei* was banned and Galileo was sent to prison. On 22 June 1633, Galileo apologized to the Church and swore not to teach that the earth moves. It is said that after the apology he muttered: "*Eppur si muove*" (indeed it moves).

- *How fast can one move?*

  It is interesting to know that no material object can move at a speed of more than 3×10^8 m/s. This is the speed at which light travels in vacuum. This is one of the main results of the 'special theory of relativity' propounded by Albert Einstein in the year 1905.

Activities

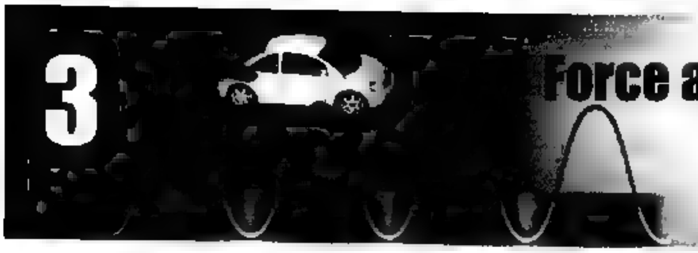
Find out the distance of your school from your house. When you go to school from your house, note down the time you take. Calculate the average speed at which you go to school. Ask several friends to do the same and prepare a chart showing average speeds of (a) a bus, (b) a bicycle, (c) a scooter and (d) a car in your city.

Watch an ant moving on the floor or a wall. Measure the distance it travels and the time it takes to travel this distance. Calculate the speed of the ant.

Cars and taxis have an odometer which measures the distance moved in kilometres. While going by a car or a taxi, note down the number of kilometres travelled after every five minutes. Draw the distance–time graph.

Identify the regions in which the road was busy or the vehicle had to stop at red lights. Identify the regions in which the vehicle travelled the fastest.

❖



3 Force and Acceleration

When the position of a body does not change with time, we say that it is at rest. If a body moves in such a way that its speed as well as its direction remains the same, it is said to move with a uniform velocity. If the speed of a body or its direction of motion changes, it is said to be accelerated. Thus, if a body at rest starts moving, it is accelerated. If a body is moving and its speed increases or decreases, it is accelerated. If a body is moving and its direction changes then also it is accelerated. How can a body at rest start moving? How can the speed of a moving body increase or decrease? How can the direction of a moving body change? These three questions can be combined into a single question. How can a body be accelerated? The answer is: A body can be accelerated by applying a force on it. Let us now discuss what a force is.

FORCE

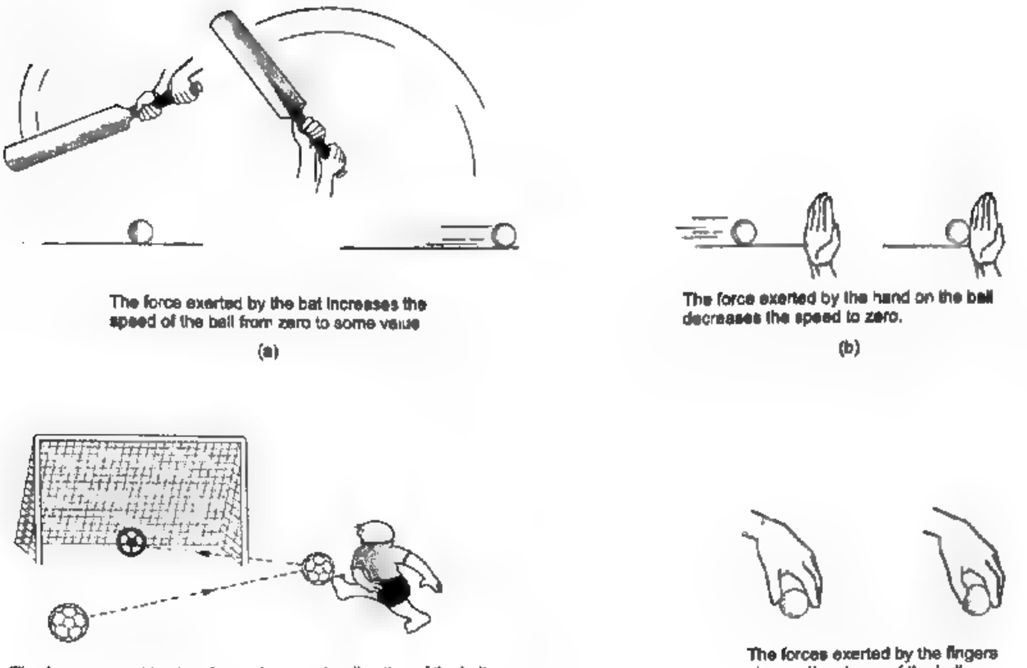
When you push a chair, it tends to move. To pluck an apple from a tree, you pull the apple towards yourself. When you push or pull an object, you exert a force on the object. You can push or pull an object in different directions. This means force has a direction. Also, you can push or pull an object gently or hard. This means force has a magnitude.

Consider a ball kept on a flat table. We can move it by pushing it or by pulling it. Now, consider the same ball moving on the table. We can increase its speed by pushing it in the direction of its motion. If we push it in the direction opposite to that of its motion, the speed will decrease. What happens if a ball is moving towards the east and you push it towards the north? The direction of the ball changes. In all these cases, we have applied a force on the ball and the ball is accelerated. So, to accelerate an object, we need to apply a force on it. In fact, we can define force as follows:

Force is that cause which produces acceleration in the body on which it acts.

Apart from accelerating an object, the shape of the object can also be changed by applying forces on it. If you take a soft rubber ball between your fingers and press the ball from two sides, the shape of the ball changes. Figure 3.1 shows some of the effects of forces. We see that a force or a set of forces acting on a body can do three things.

(a) A force or a set of forces can change the speed of a body.
(b) A force or a set of forces can change the direction of motion of a body
(c) A set of forces can change the shape of a body.



The force exerted by the bat increases the speed of the ball from zero to some value

(a)

The force exerted by the hand on the ball decreases the speed to zero.

(b)

The force exerted by the player changes the direction of the ball.

(c)

The forces exerted by the fingers change the shape of the ball.

(d)

Fig. 3.1

Resultant Force

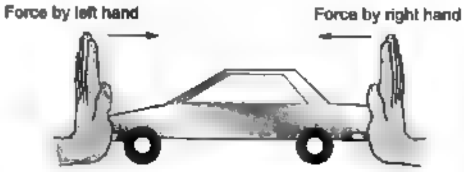
Many forces may be applied simultaneously on a body. For example, several people may jointly make an effort to move a heavy cupboard. Each person pushes it, i.e., applies a force on the cupboard. These forces together produce some acceleration in the cupboard. It is also possible that one strong man pushes the cupboard hard enough to produce the same acceleration in it.

If a single force acting on a body produces the same acceleration as produced by a number of forces, that single force is called the resultant of these individual forces.

The resultant force is also called the net force.

Balanced and Unbalanced Forces

Consider a toy car placed on a table. Suppose it can move on its wheels along the east–west direction. If you push the car towards the east with your left hand, it moves towards the east. If you push the car towards the west with your right hand, it moves towards the west. If you push the car towards the east with your left hand and simultaneously push it towards the west with your right hand, it is possible that the car will remain at rest. In this case, the two forces balance each other, so there is no acceleration.



The toy car does not move when the applied forces are equal and opposite.

Fig. 3.2

If a set of forces acting on a body produces no acceleration in it, the forces are said to be balanced. If it produces a nonzero acceleration, the forces are said to be unbalanced.

If two forces balance each other, they must be in opposite directions and have equal magnitudes.

Some Common Forces

We have considered examples of forces which are exerted by living things. However, nonliving

things can also exert forces. If we stand in a field on a stormy day, we can feel the force exerted by the wind on us. If a heavy load is put on the head of a man, the load exerts a downward force on him. When we drop a coin, it falls. In this case, the earth exerts a force on the coin to pull it down. Let us now discuss the nature of some common forces.

(a) Contact force When a body A is in contact with another body B then A can exert a force on B, and B can exert a force on A. These forces are called contact forces. Push or pull exerted by a person, force by the wind, force exerted by a load on the head of a porter, etc., are examples of contact forces.

(b) Normal force If the contact forces between two bodies are perpendicular to the surfaces in contact, the forces are called normal forces. Consider a box placed on the palm of your hand. The hand pushes the box upwards, and the box pushes the hand downwards. These forces are perpendicular to the surfaces of the hand and the box. Thus, the hand applies a normal force on the box in the upward direction, and the box applies a normal force on the hand in the downward direction.

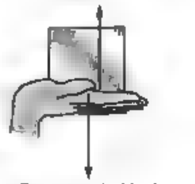


Fig. 3.3

(c) Friction Two bodies placed in contact can also exert forces parallel to the surfaces in contact. Such a force is called force of friction, frictional force or simply friction. The effect of friction is to oppose the slipping of the two bodies against each other. Friction is small if the surfaces are smooth, and it is large if the surfaces are rough.

Consider a book placed on a horizontal table. If you try to push the book gently with your finger, the book does not move. This is because the table exerts a force on the book in the direction opposite to that of your push. This force is the force of friction exerted by the table on the book. Its magnitude is equal to that of the force exerted by the finger on the book. Thus, there is no unbalanced force on the book.

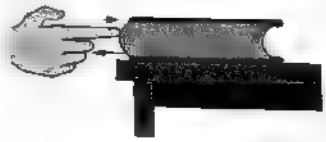

The finger pushes the book towards the right, and the table pushes it towards the left. The book does not move.

Fig. 3.4

Now, push the book slightly harder. The force exerted by the finger on the book increases, but the book still does not move. This means the force of friction exerted by the table in the opposite direction has also increased.

This shows a very interesting property of friction. The force of friction can change its value to prevent slipping of two objects against each other. However, the force of friction cannot increase beyond a certain limit. If you push the book hard enough, there will come a point when the force of friction can no longer increase to become equal to the pushing force. Then the book will start sliding on the table.

(d) Forces exerted by a spring You must have seen the spring inside a ballpoint pen. A spring is made of a coiled metallic wire. It has a definite length when it has been neither pushed nor pulled. This length is called the natural length of the spring. If the spring is pulled at the ends so that its

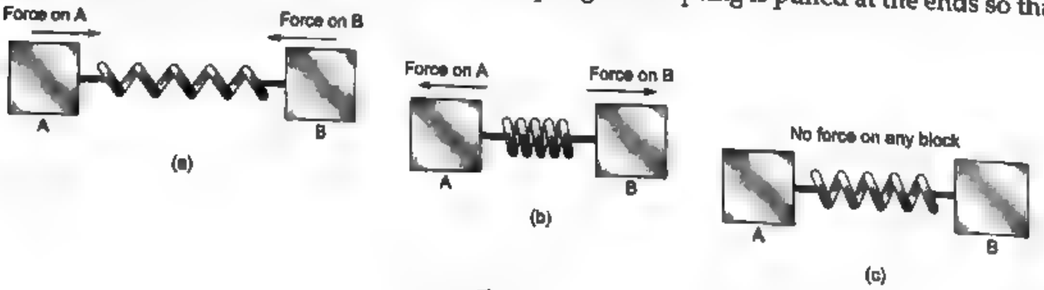


Fig. 3.5

length becomes larger than its natural length, it is called a stretched spring or an extended spring. An extended spring pulls objects attached to its ends (Figure 3.5a). If the spring is pushed at the ends, its length becomes less than its natural length. Such a spring is called a compressed spring. A compressed spring pushes objects attached to its ends (Figure 3.5b). A spring in its natural length does not exert any force on the objects attached to its ends (Figure 3.5c)

(e) Force exerted by a string When an object is tied to a string and the string is kept taut, the string pulls the object. For example, suppose you hang a bottle from a string. The string pulls the bottle in the upward direction. The magnitude of the force exerted by a string on the object tied to it is called tension.

(f) Weight The earth attracts all bodies towards its centre. The force exerted by the earth on a body is called the weight of the body. This force acts towards the centre of the earth, i.e., in the vertically downward direction. Note that this is not a contact force. Even if an object is not in contact with the earth, the earth pulls it.

If the mass of a body is m and it is placed near the surface of the earth, its weight is

$$W = mg,$$

where the value of g is approximately 9.8 m/s^2.

Unit of Force

The SI unit of force is called newton and its symbol is N. Approximately, it is the force exerted by your hand to hold a body of mass 102 g. We will give a more precise definition of the newton later.

GALILEO'S EXPERIMENTS

We have seen that an unbalanced force is needed to accelerate a body. But for long people believed that a force is needed to maintain the uniform motion of an object, and if no force acts on the object, it would come to rest Many things we see around us mislead people into believing this. Let us see one such case.

Consider a ball moving on a horizontal table with some speed. Suppose we do not do anything to the ball, and just watch. We find that its speed decreases and it stops after some distance. It may appear that the ball is slowing down although no force is acting on it. However, this is not true. Although we have not applied any force on the ball, there are other agencies applying forces on it. The table exerts a force of friction on the ball, and it is this unbalanced force that is responsible for the slowing down of the ball. If the table is made smoother, the force of friction is reduced and the ball moves through a larger distance. Imagine a situation in which we get rid of friction completely. Then the ball will continue to move without slowing down till it reaches the edge of the table and drops off.

It was Galileo who recognized that a body moves with uniform velocity if no force acts on it. Before Galileo, there was a general belief that a force is necessary to keep a body moving with uniform velocity. Galileo arrived at the correct conclusion through a series of experiments and reasonings.

One of his experiments involved observing the motion of balls rolling on an inclined plane (Figure 3.6). When a ball rolls down an inclined plane, its speed increases, whereas while rolling

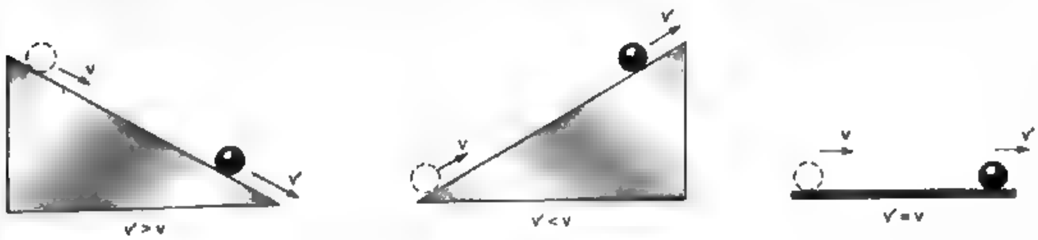


Fig. 3.6

up an inclined plane, the speed decreases. What will happen if it rolls on a horizontal plane? The case is just in between the two situations described above. The result must also be in between, i.e., its speed should remain constant. This can be explained as follows.

Moving up: speed decreases

Moving down: speed increases

Moving horizontally (neither up nor down): speed constant (neither decreases nor increases)

Galileo reached this conclusion in another way too. Consider a small ball tied to one end of a string. The other end of the string is fixed by a clamp. If the ball is pulled to one side and released, it will start swinging. Such a system is called a simple pendulum (Figure 3.7a). It is observed that when the ball is released from a height, it swings to the other side and stops momentarily on reaching almost the same height from which it was released. Galileo reasoned that the same would be true if no string is used and the ball is released on the inner surface of a smooth (frictionless) hemisphere. It will move to the other side and reach the same height before coming to rest momentarily (Figure 3.7b). What will happen if the hemisphere is replaced by a surface shown in Figure 3.7c? In order to reach the same height, the ball has to travel through a larger distance on the other side. This means, it will stop after travelling a larger distance. As we go on tilting the other side towards the horizontal, the ball has to cover larger distances to reach the same height (Figure 3.7d). If the other side is made horizontal (Figure 3.7e), it will never stop, because it will never be able to reach the same height. This means its speed will not decrease, and hence, it will travel with uniform velocity on a horizontal surface.

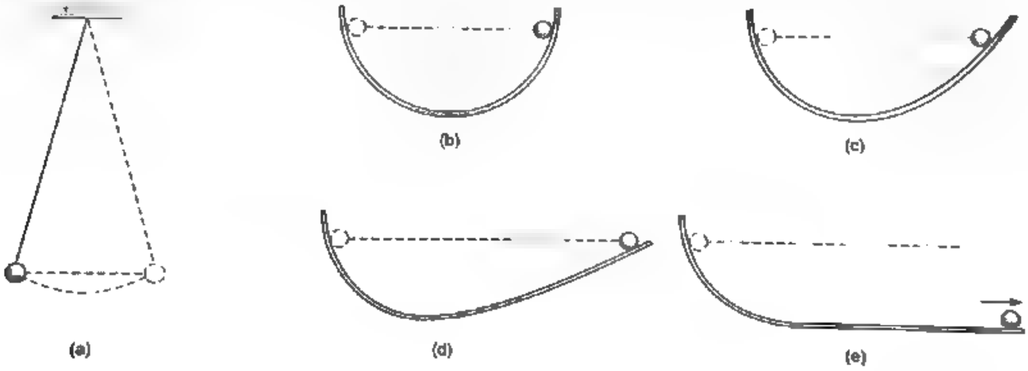


Fig. 3.7

NEWTON'S FIRST LAW OF MOTION

We have learnt so far that

(a) if a body is at rest and no unbalanced force acts on it, it remains at rest

(b) if a body is moving and no unbalanced force acts on it, it will continue to move at constant speed in a fixed direction

(c) if an unbalanced force acts on a body, the body will accelerate

These facts taken together form Galileo's law of inertia or Newton's first law of motion. The idea was suggested by Galileo and was later formulated into a law by Newton. We can state Newton's first law as follows.

➤ **Newton's First Law** ***A body at rest will remain at rest and a body in motion will remain in uniform motion unless acted upon by an unbalanced force.***

This law may also be stated as follows.

A body remains unaccelerated if, and only if, the resultant force on it is zero.

In such a case the body is said to be in equilibrium.

Inertia

We see that a body on its own does not change its state of rest or of uniform motion. If it is at rest, it remains at rest. If it is moving, it continues to move without any change in either speed or direction. Only a force can change its state of rest or its state of motion.

The inability of a body to change on its own its state of rest or of uniform motion is known as inertia.

Since the law stated above is about inertia, it is also known as Galileo's law of inertia.

Inertia may be thought of as having two forms: inertia of rest, due to which a body at rest remains at rest, and inertia of motion, due to which a moving body keeps moving without any change in its velocity.

Inertia and mass To change the velocity of a given body, one has to apply a force. Consider two bodies of unequal masses, say a football and a tennis ball. If you push the two balls with equal force for the same amount of time, both will start moving. But the football will gain a much smaller velocity than the tennis ball. Thus, the same effort applied for the same time brings about a smaller change in the velocity of the football as compared to the change in the velocity of the tennis ball. We say that the football has resisted the attempt to change its state more effectively than the tennis ball has. In other words, a football has a larger inertia than a tennis ball In general, a heavier body has greater inertia than a lighter body. This is true for inertia of rest as well as for inertia of motion. Consider two bodies—one of mass 2 kg and the other of mass 100 g—coming towards you with equal velocities. Suppose you try to stop them with your hands, one with each hand. It is possible that while the lighter one will come to rest, the heavier one may slow down only a little. So, the larger the mass, the larger is the inertia, and the smaller the mass, the smaller is the inertia. So mass is a measure of inertia

Examples of Newton's First Law

There are numerous examples in daily life which confirm Newton's first law.

(a) **Inertia of rest** If we keep a body at rest at a place, it remains there for any length of time if no force is applied on it.

(b) **Jerks while travelling** When we stand in a bus and the bus starts suddenly, we tend to fall backwards. This is because our feet are in contact with the floor of the bus and the friction at the contact is high. This force does not allow the feet to slip on the floor. The feet, therefore, move forward with the floor. The upper part of the body does not feel the forward force immediately and remains at rest for a while. So, the upper part of our body gets jerked backward.

Similarly, when the bus stops suddenly, the feet come to rest immediately, but the upper part of the body continues to move in the forward direction. So, we tend to fall forward. We also tend to fall sideways when the bus turns sharply. This is because the feet turn with the floor, but the upper part of the body continues to move for a while in the original direction.

(c) **The card-and-coin experiment** Take a smooth card (say a plastic card) and keep it on a glass (Figure 3.8). Place a coin on the card and flick the card sharply in the horizontal direction. The card flies away and the coin drops into the glass.

Fig. 3.8

Initially, there are two forces on the coin. The earth pulls the coin downwards (weight) and the card pushes it upwards (normal force). The forces balance each other and the coin remains at rest.

When we apply a horizontal force on the card, it is accelerated and it moves away. Since the friction between the card and the coin is negligible, there is no force on the coin in the horizontal direction. It remains in its original position due to inertia of rest.

When the card moves away from underneath the coin, the normal force on the coin becomes zero. The weight is the only force on the coin, and hence, it accelerates downwards to fall into the glass.

(d) Striking a pile of carrom coins Make a pile of carrom coins. Now, hit the bottom coin hard with a striker. If you do it well, the lowest coin will move away, but the rest of the pile will remain at the original position. The lowest coin moves because of the force exerted by the striker on it. However, the rest of the pile remains at its place due to inertia of rest. As the lowest coin moves very fast, any force exerted by it on the coins above it is for a very short time, which is not able to move the upper coins in the horizontal direction.

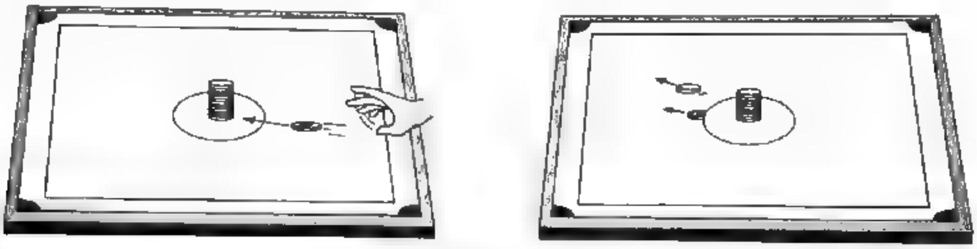


Fig. 3.9

EXAMPLE 3.1 **A body of weight 5 N is kept on a smooth horizontal table. Find the force exerted by the table on the body.**

Solution The body is at rest on the table. From Newton's first law, the resultant force on the body must be zero. Its weight is 5 N. This means the earth is attracting it by a force of 5 N. This force is in the downward direction. The table must exert a force of 5 N in the upward direction to make the resultant force zero.

EXAMPLE 3.2 **A block of weight 5 N is placed on a horizontal table. A person pushes the block from top by exerting a downward force of 3 N on it. Find the force exerted by the table on the body.**

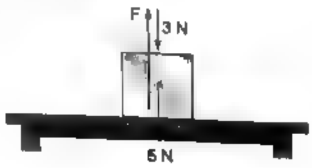


Fig. 3.10

Solution There are three forces on the body:

(a) 5 N, downward, by the earth,

(b) 3 N, downward, by the person, and

(c) F, upward, by the table.

As the body is at rest, the resultant force on it must be zero. The total downward force is

$$5\text{ N} + 3\text{ N} = 8\text{ N}$$

Hence, the upward force F should also be 8 N. So, the table exerts a force of 8 N on the body in the upward direction.

NEWTON'S SECOND LAW OF MOTION

Newton's first law tells us what a force is. Force is the cause of acceleration. We know that there are small forces and large forces. You can push a body gently or very hard. If you push a body gently, a small acceleration is produced. If you push the same body harder, a larger acceleration is produced. This gives us a way to compare numerically the strengths of two forces—apply the forces separately to the same body and measure their accelerations. A comparison of the accelerations gives an idea of the strengths of the forces If Force-1 produces an acceleration of

5 m/s^2 and Force-2 produces an acceleration of 10 m/s^2 in the same body then Force-2 must have double the magnitude of Force-1. If we denote the magnitude of force by F and acceleration by a,

$$F \propto a, \qquad \ldots \text{(i)}$$

provided that forces are exerted on the same body, i.e., the mass m is fixed.

We can think of producing the same acceleration in two objects of unequal masses. If you try to accelerate a football and a tennis ball by the same amount, the force applied on the football has to be larger than that on the tennis ball. Suppose we apply a force F on a body of 2 kg, which produces an acceleration of 5 m/s^2. To produce the same acceleration in a 4-kg body, we have to apply a force twice as strong as F. Thus, if a is fixed,

$$F \propto m. \qquad \ldots \text{(ii)}$$

Relations (i) and (ii) are summarized in Newton's second law, which states the following.

Newton's Second Law: ***The magnitude of the net force acting on a body is proportional to the product of the mass of the body and its acceleration. The direction of the force is the same as that of the acceleration.***

Thus,

$$F = kma, \qquad \ldots 3.1$$

where k is a constant.

EXAMPLE 3.3 **A force F_1 acting on a 2.0-kg body produces an acceleration of 2.5 m/s^2. Another force F_2 acting on a 5.0-kg body produces an acceleration of 2.0 m/s^2. Find the ratio F_2/F_1.**

Solution We have $F = kma$. Thus,

$$F_1 = k\,(2.0\text{ kg})\,(2.5\text{ m/s}^2), \text{ and}$$
$$F_2 = k\,(5.0\text{ kg})\,(2.0\text{ m/s}^2).$$

These equations give

$$\frac{F_2}{F_1} = \frac{5.0 \times 2.0}{2.0 \times 2.5} = 2.$$

Linear Momentum

The product of the mass of a body and its velocity is called the linear momentum of the body. Quite often we only use the word momentum for linear momentum. If m be the mass and v be the velocity of a body at some instant, the linear momentum p of the body at that instant is

$$\boxed{p = mv} \qquad \ldots 3.2$$

Besides magnitude, linear momentum also has a direction. At any instant, its direction is the same as the direction of the velocity.

Why have physicists given a separate name (momentum) to the quantity mv? This is because there are several effects which vary with mass and also with velocity, but are the same if the product mv is the same. So, the effects are controlled by the product mv. This product, therefore, has special significance. You can do the following activity to understand this.

Take two identical plastic balls, Make a hole in one of them and fill it with sand. Seal the hole with tape. Now you have two balls of different masses. Make a pile of sand on the floor and drop the lighter ball on it from a height. The ball will penetrate into the sand to some depth. Make the pile again and drop the heavier ball on it from the same height. This ball will penetrate deeper into the sand. Dropping from the same height ensures that balls strike the sand with the same velocity. So, increasing the mass increases the depth of penetration when the velocity remains the same.

Now drop one of the balls from different heights on the pile. Dropping from a larger height means the ball strikes the pile with larger velocity. So, increasing the velocity increases the depth of penetration when the mass remains the same.

If you drop the lighter ball from a greater height and the heavier ball from a lower height, it may happen that the depths of penetration become the same. This will happen when the product mv is the same.

Let us look at some other examples. Suppose someone asks you to hold a one-kilogram block on your head. You will do it readily because it is easy, and no harm will come to your head. But you will not allow the same block, moving with some velocity, to hit your head. This is because, although the mass remains the same, the momentum mv becomes large. And that can cause serious injury.

A bullet fired at a wooden board can destroy it. But a small stone of the same mass as the bullet when thrown at the board will hardly cause any damage. Although their masses are the same, the high velocity of the bullet gives it a large momentum, which causes the damage.

Newton's Second Law in Terms of Momentum

Newton had recognized the importance of momentum and had originally formulated the second law of motion in terms of change in momentum. The statement of the second law in terms of momentum can be given as follows.

➤ **Newton's Second Law** *The rate of change of momentum of an object is proportional to the net force applied on the object. The direction of the change of momentum is the same as the direction of the net force.*

Let us see how the two ways of stating the second law are equivalent. Suppose an object of mass m is moving along a straight line with a constant acceleration a. Also, suppose its velocity at time t_1 is v_1, which changes to v_2 at time t_2.

The linear momentum at time t_1 is $p_1 = mv_1$ and that at time t_2 is $p_2 = mv_2$.

The rate of change of momentum is $\dfrac{p_2 - p_1}{t_2 - t_1}$.

According to the second law,

$$\frac{p_2 - p_1}{t_2 - t_1} \propto F$$

or

$$F = k\frac{p_2 - p_1}{t_2 - t_1} \quad \text{(where } k \text{ is a constant)}$$

$$= k\frac{mv_2 - mv_1}{t_2 - t_1} = km\left(\frac{v_2 - v_1}{t_2 - t_1}\right)$$

or

$$F = kma,$$

which is the same as Equation 3.1.

Definition of Newton

Equation 3.1 gives us a way of comparing two forces. Our next task is to choose a force and call it 1 unit. Then any force may be given a definite numerical value by comparing it to the unit force.

If a force acting on a body of mass 1 kg produces an acceleration of 1 m/s^2 in it, the force is called one newton.

If we substitute $F = 1$ newton, $m = 1$ kg and $a = 1$ m/s^2 in Equation 3.1, we get $k = 1$. Equation 3.1 then becomes

$$\boxed{F = ma} \quad \text{...3.3}$$

This equation is taken to be the statement of Newton's second law in the mathematical form.

Note that the unit 'newton' is identical to kg m/s^2. This is because in $F = ma$, the right-hand side is in kg m/s^2. The newton is denoted by the symbol N. It is the SI unit of force.

With this definition of the newton, we got $k = 1$.

The equation $F = k\dfrac{p_2 - p_1}{t_2 - t_1}$ then becomes

$$\boxed{F = \frac{p_2 - p_1}{t_2 - t_1}} \qquad \text{...3.4}$$

So, the net force on a body is equal to the change in its momentum per unit time.

EXAMPLE 3.4 **Find the values of F_1 and F_2 in Example 3.3.**

Solution From $F = ma$,

$F_1 = (2.0\ \text{kg})(2.5\ \text{m/s}^2) = 5.0\ \text{kg m/s}^2 = 5.0\ \text{N}.$

$F_2 = (5.0\ \text{kg})(2.0\ \text{m/s}^2) = 10\ \text{kg m/s}^2 = 10.0\ \text{N}.$

Sports and the Second Law

Fig. 3.11

You might know that it is easier to catch a fast moving cricket ball by pulling back your arms while taking the catch. This allows you to catch the ball by applying a small force on it. If you keep your arms still as you catch the ball, you have to apply a much larger force on the ball. The durations for which the forces are applied are also different in the two cases. When you pull your arms back with the ball, you apply a smaller force for a longer time. This is because the ball stops completely (as seen by the umpire) only when your hands stop. In the second case, the ball stops almost immediately as it hits your still hands. You have to apply a larger force for a lesser time. Let us understand this on the basis of Newton's second law.

Suppose the ball is moving with a speed u. When it stops completely, its speed is $v = 0$. Suppose, in the first case, when you pull your arms back, it takes time t_1 to stop the ball. The acceleration of the ball in this period is

$$a_1 = \frac{v-u}{t_1} = \frac{0-u}{t_1} = -\frac{u}{t_1}.$$

Similarly, if the time taken to stop the ball is t_2 in the second case,

$$a_2 = -\frac{u}{t_2}.$$

The forces in the two cases are

$$F_1 = ma_1 = -\frac{mu}{t_1} \quad \text{and} \quad F_2 = ma_2 = -\frac{mu}{t_2}.$$

$$\therefore \qquad \frac{F_2}{F_1} = \frac{t_1}{t_2}.$$

As t_1 is much larger than t_2, the force F_2 is much larger than F_1. A cricket coach will, therefore, advise you to pull back your arms while taking a catch, as this would be easier.

Let us take another example. If you jump from a height on a hard floor, you can hurt your feet. This is because the feet come to rest in a very short time, and hence, the force exerted on your

feet by the floor is very higher. That is why athletes doing high jump land on sand or foam. This increases the time in which the body comes to rest. The sand gives way, allowing the body to slow down gradually. This decreases the force on the body, and the landing is comfortable.

More on Newton's First and Second Laws

If you put $F = 0$ in Equation 3.3, you get $a = 0$. This means, if no unbalanced force is applied on an object, its acceleration will be zero. If it is at rest, it will remain at rest and if it is moving, it will continue to move in the same direction with the same speed. But this is the statement of Newton's first law.

It may seem that Newton's first law is only a special case of Newton's second law. However, the first law has much deeper significance in physics, about which you will learn in higher classes.

NEWTON'S THIRD LAW OF MOTION

Newton's first and second laws deal with the effect of forces when they act on a body. The first law states that unbalanced forces produce a change in the velocity of the body and the second law gives the exact amount of force needed to produce a given acceleration. Newton's third law is a relation between the forces themselves. When a body exerts a force on another body, the other body also exerts a force on the first. We say that the two bodies *interact* with each other or an interaction takes place between the bodies. Newton's third law states:

➤ **Newton's Third Law** ***In any interaction between two bodies, the force applied by the first body on the second is equal and opposite to the force applied by the second body on the first.***

This means that forces always occur in pairs. Here are some examples. When you push a book towards the left with your hand (Figure 3.12a), the hand exerts a force on the book towards the left. According to Newton's third law, the book also exerts a force on the hand. This force is towards the right, and it has the same magnitude as the force with which you push the book.

When a porter carries a heavy load on his head (Figure 3.12b), the load pushes down on his head. The porter's head pushes the load upwards. These forces are equal in magnitude.

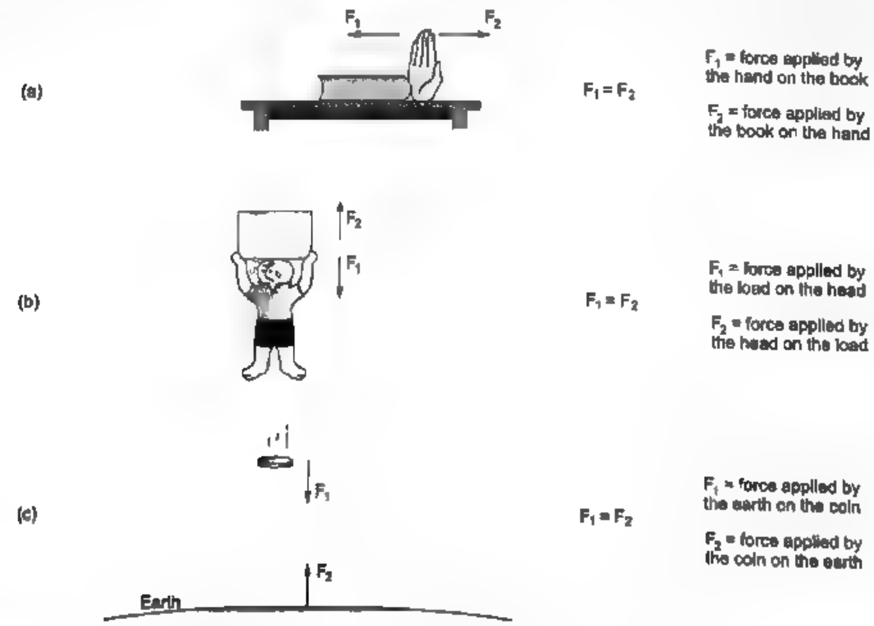


Fig. 3.12

When a coin is dropped, it falls (Figure 3.12c) This shows that the earth attracts the coin, i.e., it exerts a force on the coin in the downward direction. Also, the coin attracts the earth in the upward direction.

If you hit a wall with your fist, the wall also hits your fist with the same force, which you feel. If you slap someone on the cheek (don't try it), his cheek also 'hits' your hand with a force of equal magnitude—in a sort of tit for tat.

Action and Reaction

We have seen that whenever there is a force applied by a body A on another body B, there is also a force applied by the body B on the body A. These two forces have equal magnitudes and opposite directions. Such a pair of forces—exerted by two bodies on each other—is called an action–reaction pair. Any of the two forces may be called the action. Then the other will be the reaction. Consider, for example, a block placed on a table The block presses the table down with a force F. The table pushes the block up with an equal force F. If we say that the downward force exerted by the block on the table is the action, the upward force exerted by the table on the block is the reaction. We might equally well call the upward force exerted by the table 'action'. Then the downward force by the block will be the reaction. Study the situation shown in Figure 3.13. An arrow starting from a body represents a force on that body.

Fig. 3.13

We can restate Newton's third law in terms of action and reaction as follows.

Action and reaction are always equal and opposite.

Action and reaction act on different bodies

It should be clear from the above discussion that action and reaction are two forces that act on two different bodies. In Figure 3.13a, the action acts on the table and the reaction, on the block. In Figure 3.12a, if we say that the action acts on the book, the reaction acts on the hand. In Figure 3.12b, if the action acts on the head of the porter, the reaction acts on the load. This is true for any action–reaction pair.

Any pair of equal and opposite forces is not an action–reaction pair

Consider a book kept on a table. We have seen that the table pushes the book in the upward direction. Then why does not the book fly up? It does not fly up because there is another force on the book pulling it down. This is the force exerted by the earth on the book, which we call the weight of the book. So, there are two forces on the book—the normal force $\mathcal{N}$, upwards, applied by the table, and the force W, downwards, applied by the earth. As the book does not accelerate, we conclude that these two forces are balanced. In other words, they have equal magnitudes but opposite directions.

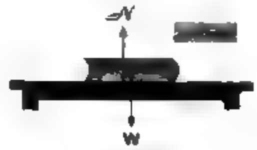


Fig. 3.14

Can we call $\mathcal{N}$ the action and W the reaction? We cannot. This is because, although they are equal and opposite, they are not forces applied by two bodies *on each other*. The force $\mathcal{N}$ is applied by the table on the book, its reaction will be the force applied by the book on the table. Weight W is the force applied by the earth on the book, its reaction will be the force applied by the book on the earth.

So, although $\mathcal{N}$ and W are equal and opposite, they do not form an action–reaction pair.

More Examples of Newton's Third Law

(a) Let us come back to the example of a cricket player catching a ball. The player has to exert a force on the ball to stop it. By Newton's third law, the ball exerts a force of equal magnitude on the hand. If he pulls his arms back while catching the ball, he has to exert a smaller force. If he keeps his arms almost still while catching, he has to exert a much larger force on the ball. In the second case, the ball also exerts a much larger force on the hands, which may hurt them.

(b) Take a small magnet and a small iron nail. Place the two at a small separation on a flat smooth surface (for example, a table with sunmica or glass top). Hold the magnet by pressing it down with your fingers. The nail moves towards the magnet and sticks to it. This shows that the magnet attracts the iron nail. Separate them and now hold down the nail. You will find that the magnet moves towards the nail and sticks to it. This shows that the nail also attracts the magnet.

(c) To start walking, you push the ground in the backward direction with your feet. By Newton's third law, the ground also pushes your feet, in the forward direction. It is this forward force acting on your feet that accelerates you in the forward direction.

(d) Suppose you are on a boat near the shore, and you jump out of the boat. As you jump, you push the boat backwards with your feet. By Newton's third law, the boat pushes you in the forward direction. Thus, you move towards the shore and the boat moves away from the shore.

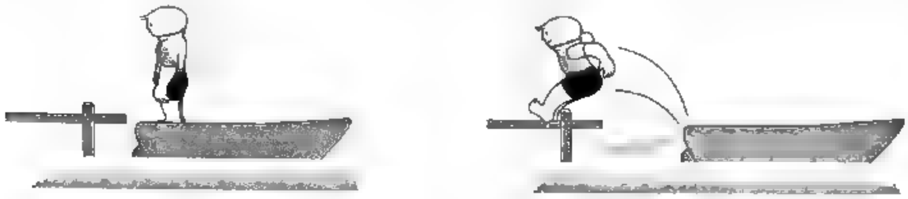


Fig. 3.15

(e) When a bullet is fired from a gun, the gun recoils. This is because the gun exerts a force on the bullet in the forward direction, and the bullet exerts an equal force on the gun in the backward direction. So, the gun moves backwards, or recoils.

(f) A jet plane accelerates by ejecting gases in the backward direction. The plane exerts a force on the gas to eject it in the backward direction. By Newton's third law, the gas exerts an equal force on the plane in the forward direction. This force accelerates the plane. The same principle is used in rockets.

(g) In a tug of war, two teams hold a rope and each team tries to pull the other towards its side (Figure 3.16). For this, each player pushes the ground in the forward direction. By Newton's third law, the ground exerts a backward force on the player. In Figure 3.16, we show the forces exerted by the players on the ground by white arrows and those exerted by the ground on the players by black arrows.

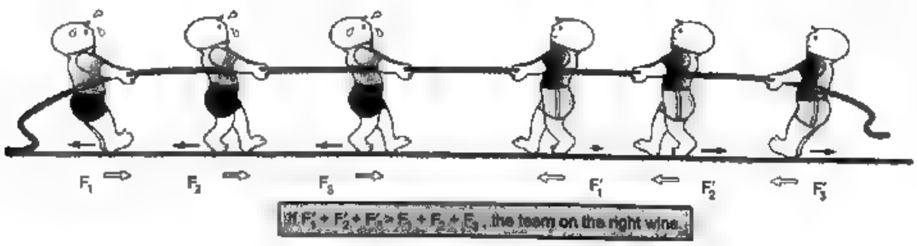


Fig. 3.16

Consider the two teams and the rope as a large body. If the two teams push the ground equally hard, the force on this combined body towards the right is equal to the force on it towards the left. The forces are balanced. But if the team on the right pushes the ground harder than the

other team, the force exerted by the ground towards the right is greater than that towards the left. The entire system (the teams and the rope) is accelerated towards the right, and the team on the right wins. Similarly, if the team on the left pushes the ground harder, it wins.

You can see the importance of the ground in deciding the contest. Suppose the ground on the left side is slippery whereas the ground on the right side is firm. It is very difficult to push on a slippery surface. So, even if the team on the left has stronger players, they won't be able to push the ground hard enough and will lose easily.

(h) Consider two balls A and B moving on a smooth, flat table (Figure 3.17). They collide, and the direction of motion of each ball changes. Also, the speeds of the balls change. When they collide, they exert forces on each other. The ball A exerts a force on B, and it is this force which changes the speed and the direction of B. Similarly, B exerts a force on A, and it is this force which changes the speed and the direction of A. By Newton's third law, these forces have equal magnitudes and opposite directions.

Fig. 3.17

Do this activity to study action–reaction forces. You will need two spring balances. When the hook of a spring balance is pulled by a force, the spring inside it gets stretched. A pointer attached to the spring reads the force on the scale of the spring balance.

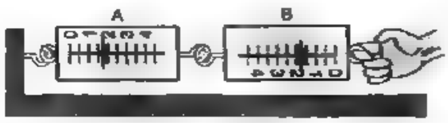


Fig. 3.18

Attach the ring of a spring balance (A) to a fixed support on a table. Pass the hook of the second sping balance (B) through the hook of A. Now, pull B by its ring. Keep applying the same amount of pull while you take readings.

Note the readings of A and B. These are equal. What do these readings show? The reading on A gives the magnitude of the force exerted on its hook, i.e., the force exerted by B. And the reading on B gives the magnitude of the force exerted on its hook, i.e., the force exerted by A. The fact that the readings are the same shows that the force exerted by A on B and that exerted by B on A have the same magnitude.

Can you say that these forces have opposite directions? Both the springs are stretched. To stretch the spring, the hook must be pulled away from the balance. So B is exerting a force on A towards the right, and A is exerting a force on B towards the left. So, the forces have opposite direction.

Consider the situation shown in Figure 3.19. Do you find anything wrong with this figure? The forces indicated in the figure form an action–reaction pair. The two goats are exerting forces on each other. By Newton's third law, these forces must have equal magnitudes. The captions 'less force' and 'more force' are, therefore, wrong.

Fig. 3.19 Spot the error.

Accelerations Produced by an Action–Reaction Pair

We know that the magnitudes of an action–reaction pair of forces are equal. However, the accelerations produced by these forces on the bodies on which they act need not be the same. This is because the masses of the two bodies may be different. Let us consider a few examples.

Suppose you drop a small stone. The earth attracts the stone and the stone attracts the earth. The magnitudes of these two forces are equal. As the mass of the stone is small, it falls with a large

acceleration (9.8 m/s²). The mass of the earth is huge. So, the acceleration produced is extremely small, which goes undetected.

While firing a bullet, a gun exerts a force on it. During this time a force of equal magnitude is exerted by the bullet on the gun. Since the bullet has a small mass, its acceleration is high and it gains a large velocity. On the other hand, because of the larger mass of the gun, its acceleration is lower, and it recoils with a much smaller velocity.

CONSERVATION OF LINEAR MOMENTUM

According to Newton's first law, a particle remains at rest or moves with a constant velocity if the total force acting on it is zero. In this case, linear momentum, which is mass times velocity, also remains constant. This is an example of a very important law of physics called the principle of conservation of linear momentum. Before stating this law, we will explain some terms.

Consider two particles A and B. We call the group of these two particles a system of particles. Particles A and B belong to this system, and any other particle is 'external' to it. The force exerted by A on B and that exerted by B on A are called internal forces. Any force on A or B by an external particle (other than A and B) is called an external force. We define the total linear momentum of the system as the sum of the linear momenta of A and B. If the two particles are moving along the same straight line, the momentum of the system is the arithmetic sum of their momenta, i.e., $m_1 v_1 + m_2 v_2$, where m_1, m_2 are the masses of the particles and v_1, v_2 are their velocities.

These ideas may be extended to a system of more than two particles. A system of particles may have any number of particles A, B, C, D, ... The forces exerted by these particles on each other are the internal forces. And the forces exerted on these particles by external particles (not in the system) are external forces. The linear momentum of the system is defined as the sum of the linear momenta of all the particles in it. The principle of conservation of linear momentum states the following.

➤ **Principle of Conservation of Linear Momentum** *If the net external force on a system of particles is zero, the linear momentum of the system remains constant.*

This means that internal forces cannot change the linear momentum of the system. Internal forces can change the momenta of different particles in the system but not the total momentum of the system. If the momentum of one particle is increased, that of some other particle must decrease to keep the total momentum of the system constant.

[illegible] **Two particles A and B of masses 20 g and 30 g respectively are at rest at a certain time. Because of the forces exerted by them on each other, the particles start moving. At a given instant, particle A is found to move towards the east with a velocity of 6 cm/s. What is the velocity of particle B at this instant?**

Solution As the particles are at rest initially, the linear momentum (mv) of each is zero. Taking the two particles together as a system, the linear momentum of the system is also zero. The forces exerted by the particles on each other are internal forces for the system and cannot change the linear momentum of the system, which remains zero during this time.

At the given instant, the linear momentum of particle A is

$$p_1 = m_1 v_1 = (20\text{ g})(6\text{ cm/s}) = 120\text{ g cm/s}.$$

The linear momentum of particle B at this instant must be −120 g cm/s to make the momentum of the system zero. Thus, particle B moves towards the west with a velocity v_2, where

$$m_2 v_2 = 120\text{ g cm/s}$$

or

$$v_2 = \frac{120\text{ g cm/s}}{m_2} = \frac{120\text{ g cm/s}}{30\text{ g}} = 4\text{ cm/s}.$$

Principle of Conservation of Linear Momentum and Newton's Laws of Motion

The principle of conservation of linear momentum follows from Newton's laws of motion. We will show this through an example.

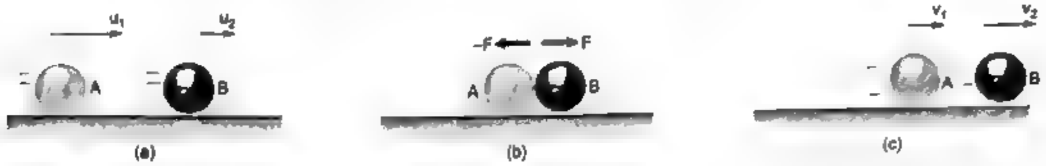


Fig. 3.20

Suppose two objects A and B of masses m_1 and m_2 respectively are kept on a horizontal table, which is so smooth that friction can be neglected.

Suppose A is made to move with a velocity u_1 and B is made to move with a velocity u_2 in the direction AB. Also, suppose that $u_1 > u_2$. The linear momentum of A is $m_1 u_1$ and that of B is $m_2 u_2$. The total linear momentum of the system is

$$p_1 = m_1 u_1 + m_2 u_2.$$

As $u_1 > u_2$, the object A will collide with B at some point of time. The two objects will remain in contact for a short itme t_0 and then they will separate. When they separate, the velocities of the objects are different from their velocities before the collision. Let the velocity of A become v_1 and that of B become v_2 after the collision (Figure 3.20c).

During the collision, A will push B towards the right, and B will push A towards the left. By Newton's third law, the magnitudes of these forces will be equal As we are taking the direction towards AB as positive, write the force on B by A as F, and the force on A by B as $-F$ (Figure 3.20b). We assume that the force F remains constant during the collision.

If we consider A and B together to be a system, the forces F and $-F$ are internal forces. Are there external forces on A and B? Consider the object A. The earth is pulling it down by the force $m_1 g$. This is an external force. The table is pushing it up by a normal force $\mathcal{N}_1$. This is also an external force. But A is neither going down nor going up. This is because $m_1 g$ and $\mathcal{N}_1$ are equal in magnitude and opposite in direction. These two forces balance each other so that the total external force on A is equal to zero. Similarly, the total external force on B is also zero. Hence the net external force on the system is zero.

The force $-F$ acting on A (due to B) produces an acceleration in it, which changes its velocity from u_1 to v_1. Using Newton's second law, the acceleration is

$$a_1 = \frac{-F}{m_1}$$

or $$\frac{v_1 - u_1}{t_0} = \frac{-F}{m_1}$$

or $$m_1 v_1 - m_1 u_1 = -F t_0. \qquad \text{...(i)}$$

The force on B is F. This produces an acceleration in B, changing its velocity from u_2 to v_2 in the same time interval t_0. Again using Newton's second law, the acceleration is

$$a_2 = \frac{F}{m_2} \quad \text{or} \quad \frac{v_2 - u_2}{t_0} = \frac{F}{m_2}$$

or $$m_2 v_2 - m_2 u_2 = F t_0. \qquad \text{...(ii)}$$

Adding Equations (i) and (ii),

$$m_1 v_1 - m_1 u_1 + m_2 v_2 - m_2 u_2 = 0$$

or $$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

or $$p_2 = p_1,$$

where p_2 is the linear momentum of the system after the interval t_0. Thus, the linear momentum of the system remained constant, although the linear momentum of each of the two particles got changed

PRESSURE

If you stand without your shoes on gravel (small, irregular stones) or stone chips, your feet hurt. The total force exerted by the gravel is equal to your weight W. Even if you stand in a nice green lawn, the force exerted by its surface on your feet is the same, W. But the soft grass of the lawn is much easier on the feet than the gravel. The reason is the difference in area over which this force is distributed. The actual area of contact between your feet and gravel is much smaller than that between your feet and grass. The feeling of discomfort or pain is more directly related to the force per unit area and not so much on the force itself

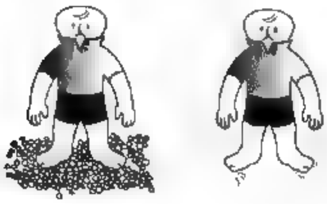


Fig. 3.21

(a) (b)

Fig. 3.22

As another example, take a brick and put it on a pile of sand, with its largest surface on the sand. Now, make another pile of sand and put another brick on it, with its smallest surface on the sand. You will see that the brick goes much deeper into the sand in the second case. In both cases, the total force by the brick on the sand is the same and is equal to the weight of the brick. Still the effect of the force is more when the area over which this force acts is smaller.

Sometimes you come across a pin whose tip is blunt. When you try to push such a pin into a bunch of papers, it does not go in. However, when you push a pin with a sharp tip with the same force, it goes into the bunch of papers easily. Once again you see that the smaller the area on which a force acts, the larger is its effect.

You can cut a potato easily using the sharp edge of a knife. But if you try to cut the same potato using the blunt edge of the knife, you may not be able to cut it, even if you apply the same or even a larger force. So, when a force acts on a smaller surface, it produces a larger effect.

All these examples show that in many cases, the force per unit area is an important quantity, and hence, it is given a separate name—pressure. If a force acts perpendicularly on a surface and is uniformly distributed over an area α of the surface, the pressure on the points over the area is defined as

$$p = \frac{F}{\alpha} \qquad \ldots 3.5$$

Unit of pressure The SI unit of force is the newton, and the unit of area is metre square. The SI unit of pressure is therefore newton per metre square, which is written as N/m^2. This unit is also called pascal, whose symbol is Pa. Thus, if 1 N of force acts on an area of 1 m^2, the pressure on the area is 1 Pa.

Thrust

The total force acting perpendicularly on a given area is sometimes called the thrust on the area.

Pressure inside a Liquid

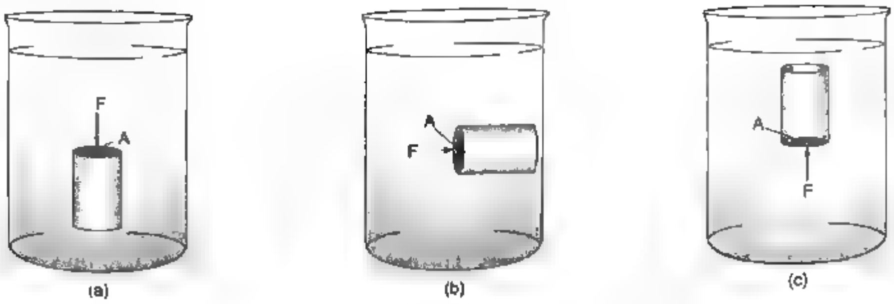


Fig. 3.23

Suppose a solid rod lies immersed in a liquid. The top surface of the rod is horizontal. This surface is shaded in the figure. The centre of this surface is at a point A in the liquid (Figure 3.23a). The liquid in contact with this surface is above it and is pressing it down. The force exerted by the liquid on this surface is thus in the downward direction. Let the area of this surface of the rod be α. This area is quite small. If the force on this surface has a magnitude F, the pressure at the point A is $p = F/\alpha$.

Now, suppose the rod is rotated so that the shaded surface becomes vertical, as shown in Figure 3.23b. The centre of the surface is still at the same point A. The liquid in contact with this surface lies to the left of this surface, and therefore, presses the surface towards the right. In other words, the direction of the force exerted on the surface is towards the right. As it happens, the magnitude of the force does not change. It is still F, and the pressure calculated at the point A using this surface area is again the same, $p = F/\alpha$.

We rotate the rod further so that the shaded surface is now at the bottom. The liquid in contact with this surface lies below it. The force exerted by the liquid on it is in the upward direction. The centre of the surface is still at the same point A, and the magnitude of the force exerted by the liquid on this surface is also the same. The pressure at A also remains the same.

The magnitude of the force exerted by a liquid on a surface inside it does not depend on how the surface is tilted. Keep it horizontal, vertical or at any angle, the force has the same magnitude. Thus the pressure at a given point in the liquid is uniquely defined. If the pressure at a point is p, and you keep a small surface of area α at that point, the force exerted by the liquid on this surface has magnitude $p\alpha$. This force acts perpendicularly on the area, pressing the surface. There are two rules that you should remember.

1. The pressure in a liquid is the same at all points at the same horizontal level.
2. As you go deeper in a liquid, the pressure increases.

Divers know this quite well. As they go deeper in water, they feel pain in the ears, owing to increased pressure.

EXAMPLE 3.6 **A cube of edge length 5 cm is placed inside a liquid. The pressure at the centre of a face is 12 Pa. Find the force exerted by the liquid on this face.**

Solution The force exerted by the liquid on the face is

$$F = p\alpha = (12\text{ Pa}) \times (5\text{ cm})^2 = (12\text{ Pa}) \times 25 \times 10^{-4}\text{ m}^2 = 0.03\text{ N}.$$

ARCHIMEDES' PRINCIPLE

When you place a rubber ball in a bucket of water, it floats. A large part of the ball remains above the surface of the water, and the rest of it remains below it. If you push the ball downwards, a

larger part of the ball goes inside water. To keep the ball at rest in this position, you have to keep on pushing it with the same force. To take the ball deeper, you have to push harder still. When you apply a large enough downward force on the ball, it gets completely immersed.

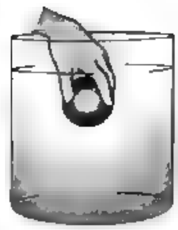


Fig. 3.24

Why do you need to push the ball downwards to keep it at rest inside water? Something must be pushing the ball upwards. To prevent the ball from moving up, you have to push it down. If you release the immersed ball, it will move up to the surface of the water. What pushes it up? It cannot be the attraction by the earth, because that force acts downwards. The only other thing that can exert a force on the ball is the water. So, we conclude that the water pushes the ball upwards.

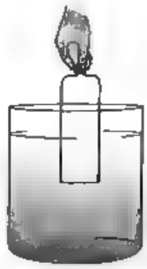
1. Fill a 1-litre plastic bottle with water and screw on its cap. Hold the bottle by the cap. You will feel some strain in your fingers because you have to apply an upward force to hold the bottle at rest.

   Now dip the bottle in a bucket of water, putting about half the bottle inside the water (Figure 3.25a). The strain on your fingers will now be less. This means that you are applying a lesser upward force to hold the bottle at rest. This is because the water exerts an upward force on the bottle. This reduces your burden.

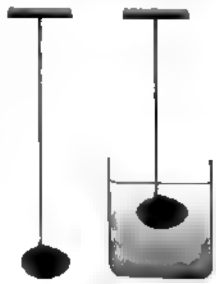
   Gradually immerse more of the bottle inside the water. As you do so, the strain on your fingers reduces further. This means that the upward force exerted by the water increases as more of the bottle gets into the water. So, when the bottle is completely immersed in water, you have to apply a very small force to hold the bottle.

2. Take a thick, long rubber band and cut it so that you have two free ends. Tie a stone at one end, and tie the other end to a fixed support (Figure 3.25b). The rubber band will get stretched. Because of this it will pull the stone upwards. This pull balances the weight of the stone.

   Now, what do you think will happen if you immerse the hanging stone in water? To check whether your answer is correct, place a jug of water below the stone so that the stone gets immersed in water. You will find that the rubber band is now stretched by a smaller amount. Why? Because the water exerts an upward force on the stone. So, the upward force needed to balance the weight of the stone reduces. Hence, the band has to exert a smaller upward upward force. Therefore, it is stretched by a smaller amount.



(a)



(b)

Fig. 3.25

The examples and activities discussed above show that when an object is partially or completely immersed in a liquid, the liquid exerts an upward force on it. What is the magnitude of the force? There is a simple rule to find it. When an object is immersed in a liquid, it has to displace some liquid to make space for itself. The force exerted by the liquid on the object is equal to the weight of the displaced liquid.

When an object is completely immersed in a liquid, the volume of the displaced liquid is the same as that of the object. When the object is partially immersed, it displaces a smaller volume of the liquid. The volume of the liquid displaced is equal to the volume of that part of the object which is inside the liquid. The upward force exerted on the object by the liquid in each case is equal to the weight of the displaced liquid. So, it is less in the second case.

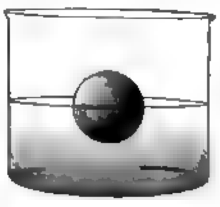


Fig. 3.26

This rule also works for solids immersed in gases. For example, consider any object in air. When the object occupied its position, it displaced air to make room for itself. Air exerts an upward force on the object, and this force has a magnitude equal to the weight of the displaced air. Liquids as well as gases can flow, and they are therefore called fluids. What we have learnt above about fluids is summarized in Archimedes' principle

Archimedes' Principle *When a solid body is partially or completely immersed in a fluid, the fluid exerts an upward force on the body, whose magnitude is equal to the weight of the displaced fluid.*

The force exerted by a fluid on a solid body immersed partially or completely in it is called the force of buoyancy, buoyant force or simply buoyancy. This force is also called the upthrust.

There is a story about how Archimedes happened to formulate this principle. He was once asked by King Heiro II to determine whether his crown was made of pure gold, or was alloyed with silver or some other lighter metal. The method for doing this occurred to him while he was taking a bath. He noticed that he displaced a volume of water while lying in his bathtub. It occurred to him that objects of the same weight but made of different materials would displace different amounts of water. Thus, from the volume of the displaced water one could tell whether the crown was made of pure gold or not (by comparing it with the volume of water displaced by the same weight of gold). It is said that Archimedes got so excited upon getting this idea that he ran out without putting his clothes on, shouting *'Eureka, Eureka'*, meaning 'I've got it'.

EXAMPLE 3.7 **A metallic sphere of mass 2.0 kg and volume 25×10^{-4} m^3 is completely immersed in water. Find the buoyant force exerted by water on the sphere. Density of water = 1,000 kg/m^3.**

Solution The sphere displaces water of volume 2.5×10^{-4} m^3 to make room for itself. The mass of the displaced water is

$$M = \rho V = \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \times (2.5 \times 10^{-4}\ \text{m}^3) = 0.25\ \text{kg}.$$

The weight of the displaced water is

$$W = Mg = (0.25\ \text{kg}) \times \left(9.8 \frac{\text{m}}{\text{s}^2}\right) - 2.45\ \text{N}.$$

Thus, by Archimedes' principle, the buoyant force exerted by water on the sphere is 2.45 N.

Floating Bodies

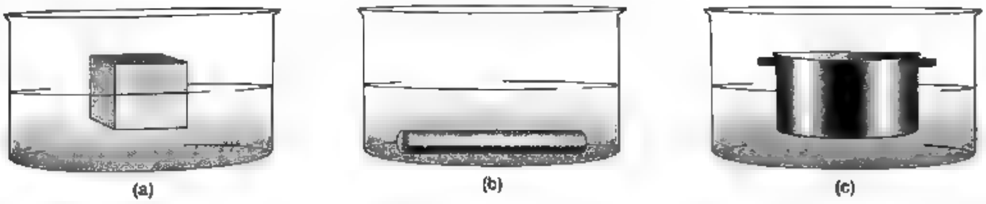


Fig. 3.27

A wooden block floats in water whereas a steel rod sinks in it. But a bowl made of steel floats in water. What decides whether a body will float or sink in a liquid? Archimedes' principle can help us answer this question. Suppose a body of weight W is immersed in a liquid and then released (Figure 3.28).

There are two forces acting on the body:

(a) the weight, W, of the body; this is the force exerted by the earth in the downward direction, and

(b) the buoyant force, B, exerted by the liquid, in the upward direction.

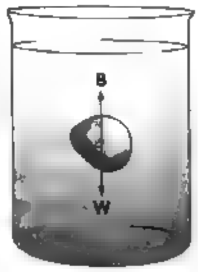


Fig. 3.28

If $W > B$, the resultant force on the body will be in the downward direction, and the body will sink. It will reach the bottom of the vessel and will stay there.

If $B > W$, the resultant force on the body will be in the upward direction. The body will come to the surface and float, with a part of it inside the liquid.

A floating body is in equilibrium, so its weight is balanced by the buoyant force. Thus,

➤ **For a Floating Body** *Weight of the displaced liquid = weight of the body*

or $$B = W.$$

EXAMPLE 3.8 **Will the sphere discussed in Example 3.7 float or sink in water?**

Solution The weight of the sphere is

$$W = Mg = (2.0\ \text{kg}) \times \left(9.8\frac{\text{m}}{\text{s}^2}\right) = 19.6\ \text{N}$$

This is larger in magnitude than the buoyant force of 2.45 N. Thus, the sphere will sink.

Condition for floatation in terms of densities

For bodies with no air space An ice cube, a wooden block, a rod, a nail, etc., are bodies that have no vacant space or air space, i.e., there is no air trapped in them. The condition for the floatation of such bodies can be expressed in terms of the densities of the bodies and the liquids concerned.

Suppose the volume of a body is V and its density is ρ.

Suppose it is immersed in a liquid of density ρ_L

The mass of the body is $M = V\rho$.

The weight of the body is $W = Mg = V\rho g$.

The volume of the displaced liquid $= V$.

The mass of this displaced liquid $= V\rho_L$.

The weight of this displaced liquid $= V\rho_L\, g$

Thus, the force of buoyancy $B = V\rho_L\, g$.

The body will come up and float if

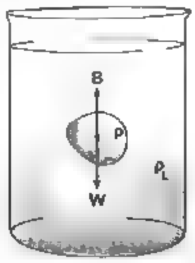


Fig. 3.29

$$B > W$$

or $$V\rho_L\, g > V\rho g$$

or $$\rho_L > \rho. \qquad \text{...3.6}$$

Thus, a body that has no air space will float only if its density is less than that of the liquid.

Ice has a density of about 900 kg/m^3 and water has density 1,000 kg/m^3. Thus, an ice cube floats in water. Iron has a density of about 8,000 kg/m^3. Thus, an iron nail sinks in water.

Relative density When a solid body (with no air space) of density ρ is dipped in a liquid of density ρ_L, it is the quantity ρ/ρ_L that decides whether the body will float or sink. This quantity ρ/ρ_L is called the relative density of the body with respect to the liquid. For such a body to float, the relative density should be less than 1.

The relative density of a body with respect to water is called its specific gravity. The density of ice is 900 kg/m^3 and that of water is 1,000 kg/m^3. The specific gravity of ice is, therefore, 900/1000 = 0.9. Note that relative density or specific gravity is a ratio of two densities. So, it is a pure number, without a unit.

For bodies with air space A small needle made of stainless steel sinks in water, but a large bowl made of stainless steel floats. Both have the same density ρ, which is much larger than the density of water. Still one floats and the other sinks. This is because a bowl can displace a much greater volume of water than the volume of steel used to make it. A covered bowl which can hold one litre of milk, can displace slightly more than one litre of water when completely immersed in water. But the volume of steel used to make this bowl is much less than one litre. In fact, one

litre of steel will weigh about 8 kg. A bowl with a capacity of 1 litre would hardly weigh a few hundred grams. So, be careful while using Equation 3.6. The shape of the body plays an important role in deciding whether it sinks or floats.

: SOLVED PROBLEMS :

EXAMPLE 1 **A force of 10 N towards the east and an unknown force *F* balance each other. Find the unknown force.**

Solution If two forces balance each other, they have to be in the opposite directions and their magnitudes have to be equal. Thus, F has a magnitude 10 N and acts towards the west.

EXAMPLE 2 **A block of mass 1.5 kg is hanging from a fixed support A through a string AB. (a) Find the force exerted by the string on the block. (b) Find the force exerted by the block on the string.**

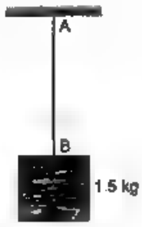


Fig. 3.W1

Solution (a) Let us see what forces are acting on the block. The earth is attracting it downwards and the string is pulling it upwards. As the block is at rest, these forces should balance each other. The force exerted by the earth, i.e., the weight of the block is

$$F = mg = (1.5\ \text{kg}) \times (9.8\ \text{m/s}) = 14.7\ \text{N}.$$

Therefore, the string pulls the block in the upward direction by a force of 14.7 N.

(b) Using Newton's third law, the force exerted by the block on the string is equal in magnitude but opposite in direction to the force exerted by the string on the block. Thus, the block pulls the string in the downward direction with a force of 14.7 N.

EXAMPLE 3 **A box of the mass 40 kg is kept on a floor. Sanjay is trying to push it from the left and Harish is trying to push it from the right. The forces exerted by them are 25 N and 35 N respectively. However, the box remains stationary on the floor. Find the force of friction acting on the box.**

Solution Apart from the forces of 25 N and 35 N exerted by Sanjay and Harish, a force of friction f acts on the box, exerted by the floor. As the box remains at rest, the resultant force on it must be zero by Newton's first law.

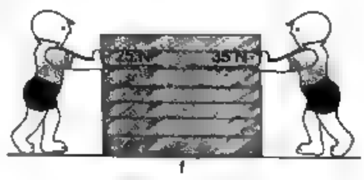


Fig. 3.W2

It is clear that resultant of the forces applied by the two people is $35\ \text{N} - 25\ \text{N} = 10\ \text{N}$, towards the left.

So, the force of friction is towards the right and its magnitude is 10 N.

EXAMPLE 4 **A force produces an acceleration of 0.5 m/s² in a body of mass 3.0 kg. If the same force acts on a body of mass 1.5 kg, what will be its acceleration?**

Solution The force is

$$F = ma = (3.0\ \text{kg}) \times (0.5\ \text{m/s}^2) = 1.5\ \text{N}.$$

The acceleration produced in the 1.5-kg body is

$$a = \frac{1.5\ \text{N}}{1.5\ \text{kg}} = 1.0\ \text{m/s}^2.$$

EXAMPLE 5 **A force produces an acceleration of 5.0 cm/s² when it acts on a body of mass 20 g. Find the force in newtons.**

Solution The force is $F = ma$

$$= (20\ \text{g}) \times (5.0\ \text{cm/s}^2) = \left(\frac{20}{1000}\ \text{kg}\right) \times \left(\frac{5.0}{100}\ \text{m/s}^2\right)$$

$$= \frac{1}{1000}\ \text{N} = 1.0 \times 10^{-3}\ \text{N}.$$

EXAMPLE **A force produces an acceleration of 2.0 m/s² in a body A and 5.0 m/s² in another body B. Find the ratio of the mass of A to the mass of B.**

Solution Let the mass of the body A be m_A and that of the body B be m_B. We have to find m_A/m_B.
We have

$$F = (m_A)(2.0\ \text{m/s}^2)$$

and $$F = (m_B)(5.0\ \text{m/s}^2).$$

Thus, $$(m_A)(2.0\ \text{m/s}^2) = (m_B)(5.0\ \text{m/s}^2)$$

or $$\frac{m_A}{m_B} = \frac{5.0}{2.0} = 2.5.$$

EXAMPLE **A force acts on a particle of mass 200 g. The velocity of the particle changes from 15 m/s to 25 m/s in 2.5 s. Assuming the force to be constant, find its magnitude.**

Solution The acceleration of the particle is

$$a = \frac{v-u}{t} = \frac{(25\ \text{m/s}) - (15\ \text{m/s})}{2.5\ \text{s}} = \frac{10}{2.5}\ \text{m/s}^2 = 4.0\ \text{m/s}^2.$$

The force is $F = ma$

$$= (200\ \text{g}) \times (4.0\ \text{m/s}^2) = \left(\frac{200}{1000}\ \text{kg}\right) \times (4.0\ \text{m/s}^2) = 0.8\ \text{N}.$$

EXAMPLE **A force of 1.0 N acts on a body of mass 10 kg. As a result, the body covers 100 cm in 4 seconds, moving along a straight line. Find the initial velocity.**

Solution The acceleration is $a = \frac{F}{m} = \frac{1.0\ \text{N}}{10\ \text{kg}} = 0.1\ \text{m/s}^2 = 10\ \text{cm/s}^2$.

We have $$s = ut + \frac{1}{2}at^2$$

or $$100\ \text{cm} = u \times (4\ \text{s}) + \frac{1}{2} \times (10\ \text{m/s}^2) \times (16\ \text{s}^2) = u \times (4\ \text{s}) + 80\ \text{cm}$$

or $$20\ \text{cm} = u \times (4\ \text{s})$$

or $$u = \frac{20\ \text{cm}}{4\ \text{s}} = 5\ \text{cm/s}.$$

EXAMPLE **A force of 0.6 N on a particle increases its velocity from 5.0 m/s to 6.0 m/s in 2 s. Find the mass of the particle.**

Solution The acceleration of the particle is

$$a = \frac{6.0\ \text{m/s} - 5.0\ \text{m/s}}{2\ \text{s}} = 0.5\ \text{m/s}^2.$$

We have $$F = ma$$

or $$m = \frac{F}{a} = \frac{0.6\ \text{N}}{0.5\ \text{m/s}^2} = 1.2\ \text{kg}.$$

EXAMPLE **A force acting on a particle of mass 200 g displaces it through 400 cm in 2 s. Find the magnitude of the force if the initial velocity of the particle is zero.**

Solution We have

$$s = ut + \frac{1}{2}at^2$$

or $$400\ \text{cm} = 0 + \frac{1}{2}a \times (2\ \text{s})^2$$

or $$800\ \text{cm} = a \times (4\ \text{s}^2)$$

or $$a = \frac{800}{4}\ \text{cm/s}^2 = 200\ \text{cm/s}^2 = \frac{200}{100}\ \text{m/s}^2 = 2\ \text{cm/s}^2.$$

The force is

$$F = ma = (200\ \text{g}) \times (2\ \text{m/s}^2) = \left(\frac{200}{1000}\ \text{kg}\right) \times (2\ \text{m/s}^2) = 0.4\ \text{N}.$$

EXAMPLE 11 **A bullet of mass 20 g moving with a speed of 120 m/s hits a thick muddy wall and penetrates into it. It takes 0.03 s to stop in the wall. Find (a) the acceleration of the bullet in the wall, (b) the force exerted by the wall on the bullet, (c) the force exerted by the bullet on the wall, and (d) the distance covered by the bullet in the wall.**

Solution (a) The velocity of the bullet as it hits the wall is $u = 120$ m/s. The velocity after 0.03 s is $v = 0$. So, using $v = u + at$,

$$0 = (120\ \text{m/s}) + a(0.03\ \text{s})$$

or $$a = -\frac{120}{0.03}\ \text{m/s}^2 = -4000\ \text{m/s}^2.$$

(b) The force exerted by the wall on the bullet is

$$F = ma$$
$$= (20\ \text{g})(-4000\ \text{m/s}^2)$$
$$= \left(\frac{20}{1000}\ \text{kg}\right)(-4000\ \text{m/s}^2) = -80\ \text{N}.$$

The negative sign shows that the force by the wall on the bullet is in a direction opposite to that of the velocity.

(c) From Newton's third law, the force exerted by the bullet on the wall is also 80 N, in direction of the velocity.

(d) The distance covered by the bullet in the wall is

$$s = ut + \frac{1}{2}at^2$$
$$= (120\ \text{m/s})(0.03\ \text{s}) + \frac{1}{2}(-4000\ \text{m/s}^2)(0.0009\ \text{s}^2)$$
$$= 3.6\ \text{m} - 1.8\ \text{m} = 1.8\ \text{m}.$$

EXAMPLE 12 **A particle of mass 0.5 kg is kept at rest. A force of 2.0 N acts on it for 5.0 s. Find the distance moved by the particle in (a) these 5.0 s, and (b) the next 5.0 s.**

Solution (a) The acceleration of the particle is

$$a = \frac{F}{m} = \frac{2.0\ \text{N}}{0.5\ \text{kg}} = 4.0\ \text{m/s}^2.$$

The distance moved by the particle in 5.0 s is

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times (4.0\ \text{m/s}^2) \times (5\ \text{s})^2 = 50\ \text{m}.$$

(b) The velocity at the end of the first 5.0 s is

$$v = u + at = 0 + (4.0\ \text{m/s}^2) \times (5.0\ \text{s}) = 20\ \text{m/s}.$$

After this, the force is withdrawn, and hence, the particle moves with uniform velocity. The distance moved in the next 5.0 s is,

$$s = ut = (20\ \text{m/s}) \times (5.0\ \text{s}) = 100\ \text{m}.$$

EXAMPLE 13 **A car of mass 1,000 kg, including everything inside it, is moving on a horizontal road with a velocity of 72 km/h. The driver applies the brakes, and the car stops in 4 seconds. Assume that the acceleration produced is constant. How much force was applied on the car during this period? Which object exerted this force?**

Solution The initial velocity is

$$u = 72\ \text{km/h} = \frac{72 \times 1000\ \text{m}}{3600\ \text{s}} = 20\ \text{m/s}.$$

The velocity after 4 s is $v = 0$.

Using $v = u + at$,

$0 = (20 \text{ m/s}) + a(4 \text{ s})$

or $a = -5 \text{ m/s}^2$.

The force on the car is

$$F = ma$$
$$= (1000 \text{ kg})(-5 \text{ m/s}^2)$$
$$= -5{,}000 \text{ N}.$$

The negative sign shows that the direction of the force is opposite to that of the velocity.

The only external force on the car that can act in the horizontal direction is the force of friction exerted by the road. So, the 5,000-N force is exerted by the road.

What is the role of the brakes then? They slow down the rotational motion of the wheels, which causes the road to apply a larger frictional force. Imagine what happens when the road is slippery and the driver applies the brakes. The car moves through a much larger distance because the road is not able to apply a large frictional force.

EXAMPLE 14 **A box of mass 50 kg is kept on a horizontal floor. When it is pushed by a horizontal force *H*, it moves with a constant velocity. If it is pushed by a horizontal force 1.5 *H*, it moves with an acceleration of 0.1 m/s². Find the value of *H*.**

Solution Since in the first case the body moves with a uniform velocity, the resultant force on it in the horizontal direction must be zero.

H + force of friction = 0

or force of friction = $-H$.

In the second case, the net horizontal force is $(1.5\,H - H) = 0.5\,H$.

Applying Newton's second law,

$$F = ma$$

or $0.5\,H = 50 \text{ kg} \times 0.1 \text{ m/s}$

or
$$H = \frac{50 \text{ kg} \times 0.1 \text{ m/s}^2}{0.5} = 10 \text{ N}.$$

EXAMPLE 15 **A force of 6.0 N produces an acceleration of 2.0 m/s² when it acts on a body A, and 3.0 m/s² when it acts on another body B. If the bodies A and B are tied together and a force of 5.0 N is applied, what will be the acceleration?**

Solution Suppose the mass of the body A is m_A and that of B is m_B. From the question,

$$6.0 \text{ N} = (m_A)(2.0 \text{ m/s}^2)$$

or
$$m_A = \frac{6.0 \text{ N}}{2.0 \text{ m/s}^2} = 3.0 \text{ kg}.$$

Similarly, $6.0 \text{ N} = (m_B)(3.0 \text{ m/s}^2)$

or $m_B = 2.0 \text{ kg}$.

When the bodies are tied together, the combined mass is 3.0 kg + 2.0 kg = 5.0 kg.

When a force of 5.0 N acts on it, the acceleration is

$$a = \frac{5.0 \text{ N}}{5.0 \text{ kg}} = 1.0 \text{ m/s}^2.$$

EXAMPLE 16 **Two carts A and B of mass 10 kg each are placed on a horizontal track. They are joined tightly by a light but strong rope C. A man holds the cart A and pulls it towards the right with a force of 70 N. The total force of friction by the track and the air on each cart is 15 N, acting towards the left. Find (a) the acceleration of the carts, and (b) the force exerted by the rope on the cart B.**

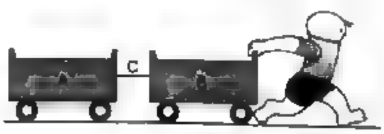


Fig. 3.W3

Solution (a) Consider the cart A, the cart B and the rope C together as one body. The man exerts a force of 70 N towards the right on this combined body. The total force of friction is 30 N towards the left. So, the resultant force is (70 N – 30 N = 40 N) towards the right. Using Newton's second law, $F = ma$, we get

$$a = \frac{F}{m} = \frac{40\ \text{N}}{20\ \text{kg}} = 2\ \text{m/s}^2,\ \text{towards the right.}$$

(b) Now, consider the cart B alone. It is also accelerating towards the right with an acceleration of $2\ \text{m/s}^2$. The rope C pulls the cart B towards the right with a force, say F_1. The force of friction on B is 15 N towards the left. So, the resultant force on it is $F_1 - 15$ N towards the right.

Using $F = ma$,

$$F_1 - 15\ \text{N} = (10\ \text{kg})(2\ \text{m/s}^2) = 20\ \text{N}$$

or

$$F_1 = 35\ \text{N}$$

Thus, the rope exerts a force of 35 N on the cart B.

EXAMPLE 17. **A force of 12 N starts acting on a body kept at rest. Find the momentum of the body at 1 s, 2 s and 5 s after the force starts acting.**

Solution We have

$$F = \frac{p_2 - p_1}{t_2 - t_1}.$$

At $t = 0$, the momentum is $p_1 = 0$. So, at a time t, its value is given by

$$F = \frac{p_2 - p_1}{t - 0}$$

or $p_2 = p_1 + Ft = Ft.$

The momentum at $t = 1$ s is $p_2 = Ft = (12\ \text{N}) \times (1\ \text{s}) = 12$ N s.

At $t = 2$ s, it is $p_2 = (12\ \text{N}) \times (2\ \text{s}) = 24$ N s.

At $t = 5$ s, it is $p_2 = (12\ \text{N}) \times (5\ \text{s}) = 60$ N s.

EXAMPLE 18 **A body of mass 300 g kept at rest breaks into two parts due to internal forces. One part of mass 200 g is found to move at a speed of 12 m/s towards the east. What will be the velocity of the other part?**

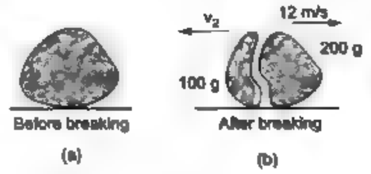


Fig. 3.W4

Solution Before it broke, the body was at rest. The linear momentum of the body was thus $p = mv = 0$.

The body breaks due to internal forces. As the external force acting on it is zero, its linear momentum will remain constant, i.e., zero.

The linear momentum of the first part is

$$p_1 = m_1 v_1 = (200\ \text{g}) \times (12\ \text{m/s}),\ \text{towards the east.}$$

For the total momentum to remain zero, the linear momentum of the other part must have the same magnitude and should be opposite in direction. It therefore moves towards the west. If its speed is v_2, its linear momentum is

$$p_2 = m_2 v_2 = (100\ \text{g}) \times v_2.$$

Thus, $(200\ \text{g}) \times (12\ \text{m/s}) = (100\ \text{g}) \times v_2$

or $v_2 = 24\ \text{m/s}.$

The velocity of the other part is 24 m/s towards the west.

EXAMPLE 19. **Two small magnets are placed at points *A* and *B* on a frictionless horizontal table. Their masses are 20 g and 12 g respectively. The magnets are released and they move on the table due to mutual attraction. When the first magnet reaches a point A_1, its speed is found to be 3.0 m/s. Find the speed of the other magnet at this instant.**

Fig. 3.W5

Solution Let the velocity of the second magnet at the given instant be v_2. Let us take the direction towards the right in the given figure as positive. Let us consider the system of the two magnets.

There is no net external force on the system. The magnets attract each other, but these forces are internal. This is because our system contains both the magnets. The total linear momentum of the system should therefore remain constant.

Initially, both the magnets are at rest. Hence, the linear momentum of each magnet is zero, and therefore, the linear momentum of the system is also zero.

At the given instant, the velocity of the first magnet is 3.0 m/s. Its linear momentum is

$$p_1 = m_1 v_1 = (20 \text{ g})(3.0 \text{ m/s}) = 60 \text{ g m/s}.$$

The linear momentum of the second magnet is $p_2 = m_2 v_2 = (12 \text{ g})\, v_2$.

Total linear momentum of the system is $p = p_1 + p_2 = (60 \text{ g m/s}) + (12 \text{ g})\, v_2$.

As the total linear momentum of the system does not change, it should be zero. Thus,

$$60 \text{ g m/s} + (12 \text{ g})\, v_2 = 0$$

$$v_2 = \frac{-60 \text{ g m/s}}{12 \text{ g}} = -5 \text{ m/s}.$$

The negative sign shows that the second magnet moves towards the left. Its speed is 5 m/s.

EXAMPLE 20 **Two toy cars A and B are moving towards each other on a horizontal surface. The car A has mass of 60 g and moves towards the right with a speed of 60 cm/s. The car B has a mass of 100 g and moves towards the left with a speed of 20 cm/s. The two cars collide and get stuck to each other. With what velocity will they move after the collision?**

Fig. 3.W6

Solution Let us take the direction towards the right as the positive direction. The linear momentum of A before the collision is

$$p_1 = m_1 v_1 = (60 \text{ g})\ (60 \text{ cm/s}) = 3600 \text{ g cm/s},$$

and that of B is

$$p_2 = m_2 v_2 = (100 \text{ g})(-20 \text{ cm/s}) = -2000 \text{ g cm/s}.$$

The total momentum of A and B before the collision is

$$p_1 + p_2 = (3600 - 2000) \text{ g cm/s} = 1600 \text{ g cm/s}.$$

After the collision, the two cars get stuck to each other and move together. Let the velocity just after the collision be v. The total momentum of A and B after the collision is

$$m_1 v + m_2 v = (60 \text{ g})v + (100 \text{ g})v = (160 \text{ g})v.$$

As the total linear momentum is the same before and after the collision,

$$1600 \text{ g cm/s} = (160 \text{ g})v$$

or $$v = 10 \text{ cm/s}.$$

As v comes out to be positive, the cars move together towards the right.

EXAMPLE 21 **The velocity–time graph of a particle of mass 50 g moving in a fixed direction is shown in Figure 3.W7. Find the force on the particle.**

Solution The velocity–time graph of the particle is a straight line. Thus, the particle moves with a uniform acceleration. From the graph, we see that the velocity at $t = 0$ is $u = 0$, and that at $t = 4$ s is $v = 2$ m/s.

The acceleration is

$$a = \frac{v-u}{t} = \frac{(2.0 \text{ m/s}) - 0}{4 \text{ s}} = 0.5 \text{ m/s}^2.$$

The mass of the particle is $50 \text{ g} = 50 \times 10^{-3}$ kg

Thus, $F = ma = (50 \times 10^{-3} \text{ kg}) \times (0.5 \text{ m/s}^2)$

$= 25 \times 10^{-3}$ N.

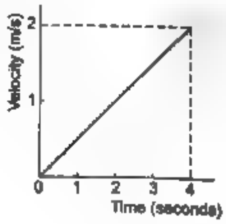


Fig. 3.W7

EXAMPLE 22 **A force of 16 N is distributed uniformly on one surface of a cube of edge 8 cm. Find the pressure on this surface.**

Solution Area of one surface of the cube is

$$\alpha = (8\text{ cm})^2 = 64 \times 10^{-4}\text{ m}^2.$$

The pressure is $$p = \frac{F}{\alpha} = \frac{16\text{ N}}{64 \times 10^{-4}\text{ m}^2} = \frac{10^4}{4}\text{N/m}^2 = 2{,}500\text{ Pa}.$$

EXAMPLE 23 **A cubical block of copper is immersed completely in water. Each edge of the block is 1 cm in length. Find the buoyant force acting on the cube. The density of water = 1,000 kg/m^3.**

Solution The volume of the block is $(1\text{ cm})^3 = 10^{-6}\text{ m}^3$.

The block displaces this much volume of water when immersed completely in water. The mass of this water is

$$M = (\text{volume}) \times (\text{density}) = (10^{-6}\text{ m}^3) \times (1000\text{ kg/m}^3) = 10^{-3}\text{ kg}.$$

Weight of this water $= Mg = (10^{-3}\text{ kg}) \times (9.8\text{ m/s}^2) = 9.8 \times 10^{-3}\text{ N}$.

Thus, the buoyant force on the cube is 9.8×10^{-3} N.

EXAMPLE 24 **A body (without air space) of mass 150 g and volume 250 cm^3 is put in water. Will it float or sink?**

Solution The density of the body is $\rho = \frac{M}{V} = \frac{150\text{ g}}{250\text{ cm}} = 0.6\text{ g/cm}^3$.

This is less than the density of water, which is 1 g/cm^3. Thus, the body will float.

• POINTS TO REMEMBER •

- *Force*

  Force is that cause which produces acceleration in the body on which it acts.

  A force or a set of forces can (a) change the speed of the body, (b) change the direction of motion of the body, and (c) change the shape of the body.

  If a single force acting on a body produces the same acceleration as produced by a number of forces, this single force is called the resultant or net of the individual forces.

  The SI unit of force is the newton, denoted by the symbol N.

- *Balanced and unbalanced forces*

  If a set of forces acting on a body produces no acceleration in it, the forces are called balanced. If it produces a nonzero acceleration, the forces are said to be unbalanced.

- *Some common forces*

  Friction is a force exerted parallel to two surfaces in contact. The effect of friction is to oppose slipping of the two surfaces against each other.

  A stretched spring pulls the bodies connected to its ends. A compressed spring pushes the bodies connected to its ends.

  A string always pulls an object tied at its end. The magnitude of the force of the pull is called the tension in the string.

  The force by which the earth attracts a body is called the weight of the body. It is equal to the mass of the body multiplied by the acceleration due to gravity ($W = mg$).

- *Newton's laws of motion*

  First law A body at rest will remain at rest and a body in motion will remain in uniform motion unless acted upon by an unbalanced force.

  Second law The net force acting on a body is proportional to the product of the mass of the body and its acceleration.

  Third law In any interaction between two bodies, the force applied by the first body on the second is equal and opposite to the force applied by the second body on the first.

- *Definition of newton*

  If a force acting on a 1-kg mass produces an acceleration of 1 m/s^2 in it, the force is called one newton.

- *Linear momentum*

  The product of the mass of a body and its velocity is called the linear momentum of the body.

  The net force on a body is equal to change in its momentum per unit time.

- *Conservation of linear momentum*

  If the net external force acting on a system of particles is zero, the total linear momentum of the system remains constant.

- *Pressure*

  If a force F, acting perpendicularly on a surface, is uniformly distributed over an area α, the pressure at the points in this area is $p = F/\alpha$.

  The SI unit of pressure is newton/metre^2, also called pascal, whose symbol is Pa.

- *Buoyancy*

  When a solid is partially or completely immersed in a fluid (liquid or gas) the fluid exerts an upward force on the solid. This force is called the force of buoyancy, buoyant force or upthrust.

- *Archimedes' principle*

  When a solid body is partially or completely immersed in a fluid, the fluid exerts an upward force on the body, whose magnitude is equal to the weight of the displaced fluid.

- *Flotation*

  A solid floats in a liquid if the force of buoyancy has the same magnitude as the weight of the floating body.

- Density of water = 1,000 kg/m^3 = 1 g/cm^3.

- *Mathematical equations*

  (a) $F = ma$ — F = resultant force, a = acceleration in the body, m = mass of the body

  (b) $F = \dfrac{p_2 - p_1}{t_2 - t_1}$ — p_1 = momentum at time t_1, p_2 = momentum at time t_2.

• EXERCISES •

A. *Objective Questions*

I. *Pick the correct option.*

1. An unbalanced force acts on a body. The body
   (a) must remain at rest
   (b) must move with uniform velocity
   (c) must be accelerated
   (d) must move along a circle

2. If a body is not accelerated,
   (a) no force acts on it
   (b) no unbalanced force acts on it
   (c) the resultant force is not zero
   (d) a single force acts on it

3. If no force acts on a body, it will
   (a) get deshaped
   (b) move with increasing speed
   (c) either remain at rest or move in a straight line
   (d) break

4. By applying a force of 1 N, one can hold a body whose mass is approximately equal to
   (a) 100 mg (b) 100 g (c) 1 kg (d) 10 kg

5. The force of friction between two bodies is
   (a) parallel to the contact surface
   (b) perpendicular to the contact surface
   (c) inclined at 30° to the contact surface
   (d) inclined at 60° to the contact surface

6. A coin flicked across a table stops because
   (a) no force acts on it
   (b) it is very heavy
   (c) the table exerts a frictional force on it
   (d) the earth attracts it

7. The speed of a falling body increases continuously. This is because
   (a) no force acts on it
   (b) it is very light
   (c) the air exerts a frictional force on it
   (d) the earth attracts it

8. Which of the following has the largest inertia?
   (a) A pin
   (b) An inkpot
   (c) Your physics book
   (d) Your body

9. When a bus starts suddenly, the passengers standing on it lean backwards in the bus. This is an example of
   (a) Newton's first law
   (b) Newton's second law
   (c) Newton's third law
   (d) none of Newton's laws

10. A force of a given magnitude acts on a body. The acceleration of the body depends on the
    (a) mass of the body (b) volume of the body
    (c) density of the body (d) shape of the body

11. If a constant force acts on a body initially kept at rest, the distance moved by the body in time t is proportional to
    (a) t (b) t^2 (c) t^3 (d) t^4

12. The momentum of a body of given mass is proportional to its
    (a) volume (b) shape (c) speed (d) colour

13. The mass and speed of four bodies are

Body	Mass	Speed
A	1 kg	10 m/s
B	2 kg	9 m/s
C	3 kg	8 m/s
D	4 kg	7 m/s

    The body with the largest magnitude of momentum is
    (a) A (b) B (c) C (d) D

14. The principle of conservation of linear momentum states that the linear momentum of a system
    (a) cannot be changed
    (b) cannot remain constant
    (c) can be changed only if internal forces act
    (d) can be changed only if external forces act

15. Action–reaction forces
    (a) act on the same body
    (b) act on different bodies
    (c) act along different lines
    (d) act in the same direction
16. Consider a porter standing on a platform with a suitcase which presses his head with a force of 200 N. Take this force as action. The reaction force is exerted by
    (a) the head on the suitcase
    (b) the earth on the suitcase
    (c) the earth on the porter
    (d) the suitcase on the earth
17.

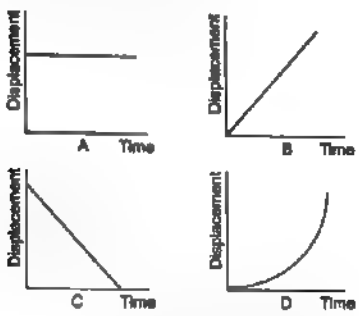


Fig. 3.E1

Figure 3.E1 shows the displacement–time graphs for the four particles A, B, C, and D. An unbalanced force is acting on the particle
    (a) A (b) B (c) C (d) D
18. Pascal is a unit of
    (a) pressure (b) force
    (c) linear momentum (d) energy
19. The buoyant force on a body acts in a
    (a) vertically downward direction
    (b) vertically upward direction
    (c) horizontal direction
    (d) direction between the horizontal and the vertical
20. A body floats in a liquid if the buoyant force is
    (a) zero (b) greater than its weight
    (c) less than its weight (d) equal to its weight

II. *Mark the statements true (T) or false (F).*

1. The speed of a particle remains constant. This means that no unbalanced force acts on it.
2. No unbalanced force acts on a particle. The speed of the particle must remain constant.
3. A spring can pull an object as well as push an object.
4. A string can pull an object as well as push an object.
5. A particle attracts the earth with a force equal to the weight of the particle.
6. A ball moving on a horizontal surface stops because of the force of friction.
7. It is easier to catch a fast-moving ball with the arms kept nearly still.
8. A particle starts from rest under the action of a constant force. The graph of distance versus time is a straight line
9. Action and reaction forces act on the same object.
10. Any pair of equal and opposite forces forms an action–reaction pair.
11. The pressures at all points in a liquid at the same horizontal plane are equal.
12. After diving into a swimming pool, as one moves up, the pressure of water increases.
13. Pascal and N/m^2 represent the same unit.
14. Pressure has magnitude as well as direction.

III. *Fill in the blanks.*

1. To accelerate a body, a ...... must act on it.
2. A ...... spring pushes the objects attached to its ends.
3. The force exerted by the earth on a body is called ...... of the body.
4. If a body A slips over another body B that is kept fixed, the force of friction on A is ...... to the direction of motion of A.
5. A body kept at rest will remain ...... if no unbalanced force acts on it.
6. The force exerted by the earth on a body is always in the ...... direction.
7. The force needed to produce an acceleration of $1\ m/s^2$ in a body of mass 1 kg is called ....
8. The forces of action and reaction have ...... magnitudes but ...... directions.
9. The linear momentum of a system remains constant if no ...... force acts on it.
10. Buoyancy is equal in magnitude to the weight of the ...... liquid.

B. *Very-Short-Answer Questions*
Answer the following in one word or maximum one sentence.

1. Under what conditions does a spring exert forces on objects attached to its ends?
2. If a ball is moving on a frictionless horizontal surface and no forces are applied on it, will its speed decrease, increase or remain constant?
3. The earth attracts an apple with a force of 1.5 N. Taking this as an action force, how much is the reaction force? Who exerts this reaction force? On which body does this reaction force act?
4. A coin falls towards the earth because the earth attracts the coin. Does the coin also attract the earth?
5. Can the internal forces acting among the parts of a system change the linear momentum of the system?
6. Does the magnitude of the force exerted by a liquid on a surface depend on whether the surface is horizontal, vertical or at some other angle?

C. *Short-Answer Questions*

Answer the following questions in about 30–40 words.

1. Define balanced and unbalanced forces.
2. Give three examples of contact forces.
3. Can nonliving objects exert a force? If yes, give two examples.
4. What is the law of inertia? Which scientist formulated it?
5. State Newton's three laws of motion.
6. Give an example to demonstrate Newton's third law of motion.
7. Suppose you are standing with your friend. Suddenly, you push your friend and he falls down. According to Newton's third law, the friend has also pushed you with a force of equal magnitude. Why did you not fall down?
8. State the principle of conservation of linear momentum.
9. Define pressure at a point.
10. State Archimedes' principle.
11. Give the relation between the buoyant force and the weight of a body for a floating body.

D. *Long-Answer Questions*

Answer the following questions in not more than 70 words

1. What are the effects a set of forces can produce?
2. Define balanced forces. Can two forces acting along perpendicular directions be balanced?
3. What do you understand by a resultant force? Explain with an example.
4. Explain the meaning of inertia. Give appropriate examples to explain the inertia of rest and that of motion.
5. Passengers tend to fall backwards when a bus suddenly starts. Explain on the basis of Newton's first law.
6. When you jump on a concrete surface, your feet hurt more than when you jump on sand. Explain on the basis of Newton's laws.
7. When you pull your arms back while catching a fast-moving cricket ball, the chances of hurting your hand are low. Explain this on the basis of Newton's laws.
8. State Newton's three laws of motion. Give one example each to explain the first and the third laws.
9. If we release a ball from our hands, it starts moving. In order to keep the ball at rest, we have to apply a force on it by holding it. Explain why this is not a violation of Newton's first law of motion.
10. Give an example where two equal and opposite forces do not form an action–reaction pair.
11. What do you understand by external and internal forces?
12. Prove the principle of conservation of linear momentum for two bodies moving along the same line and colliding.

E. *Numerical Problems*

1. A boy is wearing a shirt of mass 150 g. How much force is he exerting on the dress? Do not forget to state the direction.
2. Your physics book has a mass of 400 g. It is kept on a horizontal table. Taking $g = 10\ \mathrm{m/s^2}$, find the force (both magnitude and direction) exerted by
   (a) the table on the physics book
   (b) the physics book on the table
   (c) the earth on the physics book
   (d) the physics book on the earth.
3. How much force is needed to produce an acceleration of $16\ \mathrm{cm/s^2}$ in a body of mass 250 g?
4. A force of 10 N acts on a particle of mass 0.4 kg. Find the acceleration of the particle.
5. A body of mass 1 kg is kept at rest. A constant force of 6.0 N starts acting on it. Find the time taken by the body to move through a distance of 12 m.
6. The velocity of a particle of mass 150 g changes from 8 m/s to 12 m/s in two seconds. Assuming that a constant force acts on it, find the magnitude of the force.
7. A force of 4.0 N acts on a body of mass 2.0 kg for 4.0 s. Assuming the body to be initially at rest, find
   (a) its velocity when the force stops acting,
   (b) the distance covered in 10 s after the force starts acting.
8. A coin of mass 20 g is pushed on a table. The coin starts moving at a speed of 25 cm/s and stops in 5 seconds. Find the force of friction exerted by the table on the coin.
9. When a body is dropped from a height, it falls with an acceleration of $10\ \mathrm{m/s^2}$. If its mass is 250 g, how much force is exerted on it downwards? Who exerts this force on the body?
10. A feather of mass 20 g is dropped from a height. It is found to fall down with a constant velocity. What is the net force acting on it?
11. When a horizontal force P acts on a cart of mass 20 kg, it moves with a uniform velocity on a horizontal floor. When a force of $1.2\ P$ acts on the cart, it moves with an acceleration of $0.05\ \mathrm{m/s^2}$. Find the value of P.
12. Figure 3.E2 shows the velocity–time graph for a particle moving in a fixed direction. (a) Find the acceleration of the particle. (b) If the mass of the particle is 200 g, what is the force acting on it?

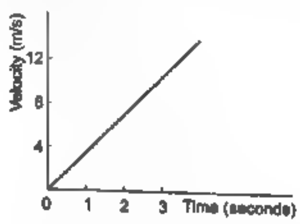


Fig. 3.E2

13. A force produces an acceleration of 1.5 m/s^2 in a disk. Three such disks are tied together and the same force is applied on the combination. What will be the acceleration?
14. A constant force of 12 N acts on a body for 4 s. Find the change in the linear momentum of the body
15. Two particles A and B of mass 10 g and 20 g respectively fall vertically. At a given time, the speed of particle A is 12 m/s and that of B is 15 m/s. Find the total linear momentum of the system of the two particles.
16. Two bodies A and B of mass 150 g and 250 g respectively are approaching each other. Both particles have a speed of 3 m/s. Find the magnitude of the total linear momentum of the system of the two particles.
17. A boy weighing 30 kg is riding a bicycle weighing 50 kg. If the bicycle is moving at a speed of 9 km/h towards the west, find the linear momentum of the bicycle–boy system in SI units.
18. A block of mass 120 g moves with a speed of 6.0 m/s on a frictionless horizontal surface towards another block of mass 180 g kept at rest. They collide and the first block stops. Find the speed of the other block after the collision.
19. A ball of mass 100 g and another ball of 120 g moves towards each other with speeds 6 m/s and 5 m/s respectively. If they stick to each other after colliding, What would be the velocity of the combined mass after the collision?
20. A cart of mass 50 kg is moving on a straight track with a speed of 12 m/s. A mass of 10 kg is gently put into the cart What will be the velocity of the cart after this?
21. A force of 12 N is uniformly distributed over an area of 120 cm^2. Find the pressure in pascals.
22. How much force should be applied on an area of 1 cm^2 to get a pressure of 12 Pa?
23. A block weighing 1.0 kg is in the shape of a cube of length 10 cm. It is kept on a horizontal table. Find the pressure on the portion of the table where the block is kept.
24. A body of volume 50 cm^3 is completely immersed in water. Find the force of buoyancy on it.
25. A metallic sphere of radius 2.0 cm is completely immersed in water. Find the force of buoyancy on it.
26. A body of mass 2.0 kg and density 8,000 kg/m^3 is completely immersed in a liquid of density 800 kg/m^3 Find the force of buoyancy on it.
27. A body of mass 2.0 kg floats in a liquid. What is the buoyant force on the body?

✦

• ANSWERS •

A. *Objective Questions*

I.	**1.** (c)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (a)
	6. (c)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (a)
	11. (b)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (b)
	16. (a)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (d)
II.	**1.** F	**2.** T	**3.** T	**4.** F	**5.** T
	6. T	**7.** F	**8.** F	**9.** F	**10.** F
	11. T	**12.** F	**13.** T	**14.** F	

E. *Numerical Problems*

1. 1.47 N upwards
2. (a) 4 N upwards (b) 4 N downwards (c) 4 N downwards (d) 4 N upwards
3. 0.04 N **4.** 25 m/s^2 **5.** 2 s
6. 0.30 N **7.** (a) 8 m/s (b) 64 m
8. 0.001 N **9.** 2.5 N, earth **10.** zero
11. 5 N **12.** (a) 4 m/s^2 (b) 0.8 N
13. 0.5 m/s^2 **14.** 48 kg m/s
15. 0.42 kg m/s downwards **16.** 0.30 kg m/s
17. 200 kg m/s **18.** 4 m/s **19.** 0
20. 10 m/s **21.** 1,000 Pa **22.** 1.2×10^{-3} N
23. 980 Pa **24.** 0.49 N **25.** 0.328 N
26. 1.96 N **27.** 19.6 N

✦

• POSTSCRIPT •

Activities

- Take a spring balance. There is a scale marked in kilograms and a pointer near the zero of the scale (Figure 3.A1). Hang the balance from a firm support and pull it with your hand. This will shift the pointer on the scale. Note the reading of the scale at the position of the pointer. Multiply the reading by ten. This gives roughly the force in newtons applied by your hand on the spring balance.

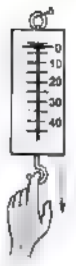
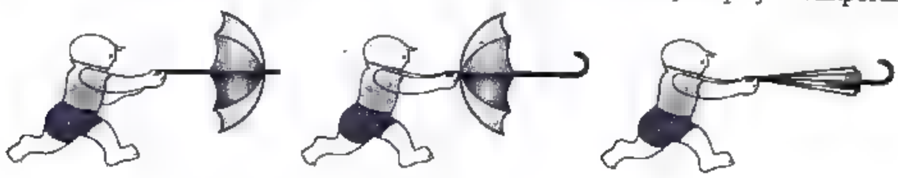


Fig. 3.A1

- Take a toy car with a spring–key system. Place a smooth plastic card (a good-quality greeting card will do) on the floor. Wind up the car and place it on the plastic card. Watch the card. You will find that the card has moved in the direction opposite to the car. This is because of Newton's third law. The card pushed the car in the forward direction and the car pushed the card in the backward direction.
- Take an umbrella and go to a field. Open the umbrella and keep it in front of you (Figure 3.A2a). Run as fast as you can. Now turn the umbrella round and hold it as shown in Figure 3.A2b. Run as fast as you can. Now close the umbrella and keep it in front of you (Figure 3.A2c). Run as fast as you can.

In which case did you run the fastest? In which case did you run the slowest? Think of the reasons and explain them to your friend. To give you a hint, the force exerted by air plays an important role.



(a) (b) (c)

Fig. 3.A2

❖

Gravitation

An apple falling from a tree is a common sight in apple-growing regions. Sometimes a thing that is very common may spark a great idea in a great mind. It is said that a falling apple led Isaac Newton to realize that the earth attracts all objects towards its centre. Newton generalized this idea and said that not only the earth, but every object in the universe attracts every other object. This force of attraction between two objects is called the force of gravitation or gravitational force

THE UNIVERSAL LAW OF GRAVITATION

The magnitude and direction of the gravitational force between two particles are given by the universal law of gravitation, which was formulated by Newton.

Universal Law of Gravitation *The gravitational force of attraction between any two particles is directly proportional to the product of the masses of the particles and is inversely proportional to the square of the distance between the particles. The direction of the force is along the line joining the two particles.*

Let A and B be two particles of masses m_1 and m_2 respectively (Figure 4.1). Let the distance $AB = r$. By the law of gravitation, the particle A attracts the particle B with a force F, such that

Fig. 4.1

$F \propto m_1 m_2$ for a given separation between the particles

and $F \propto \frac{1}{r^2}$ for a given pair of particles.

So, $F \propto \frac{m_1 m_2}{r^2}$

or
$$F = G\frac{m_1 m_2}{r^2} \quad \text{...4.1}$$

Here G is a constant known as the universal constant of gravitation. The value of G was

experimentally measured in the laboratory by Cavendish, long after Newton's death. This value is

$$G = 6.67 \times 10^{-11} \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}.$$

Equation 4.1 is applicable for particles. However, it can also be used for larger spherical bodies which have uniform density in all directions. In that case r is the distance between their centres.

Unit of G We can easily find the unit of G from Equation 4.1. We have

$$G = \frac{Fr^2}{m_1 m_2}.$$

As the units of F, r, m_1 and m_2 are N, m and kg respectively, the unit of G will be

$$\frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}$$

The constant G is universal in the sense that it does not depend on anything. Whether you consider two particles in your bedroom or in your classroom, in India or in Japan, on the earth or on Jupiter, the gravitational force of attraction will be given by Equation 4.1, using the same value of G. Similarly, whether we make the measurement today or thousands of years from now, the same value of G will be used for finding the gravitational force of attraction between two particles. So, G is independent of the nature of particles, the place where they are kept and the time at which the force is considered. It is, therefore, a 'universal' constant.

Can you think of some other universal constants? The charge on an electron is one. Think of others.

Development of Our Knowledge about Gravitation

Our knowledge about gravitation did not develop only as a result of Newton observing a falling apple. Some thinkers and scientists before Newton already had some idea about gravitational force. Indian scholars had been using the term *gurutva akarshan* meaning 'attraction by a massive body' from a time much before Newton. The study of the motion of the planets also helped in understanding the nature of this force. More than a thousand years before Newton, Aryabhatta wrote a book called *Aryabhatiya*, in which he described the motion of the moon and the planets. Johannes Kepler, who lived before Newton, formulated three laws governing the motion of the planets. From Kepler's work it is possible to show that gravitational force is proportional to $1/r^2$. Newton most certainly was influenced by the work of people like Kepler. But it was he who was able to formulate the equation which gives how much gravitational force is exerted between two bodies at a given distance.

While Kepler's laws describe how the planets move around the sun, Newton's laws of motion can be used to learn about the force which causes a planet's motion. In our discussion here, we shall assume that the planets move in circular orbits. So, first let us learn a bit about the force which causes a body to move along a circular path.

Force Required for Circular Motion

Suppose you tie a stone at the end of a string and whirl it around in a circle (Figure 4.2). As the stone moves, the direction of its velocity changes continuously, as shown by the arrows in the figure. This means that it has acceleration. From Newton's second law of motion, a force must be the cause of this acceleration. Who exerts this force, and what is its direction?

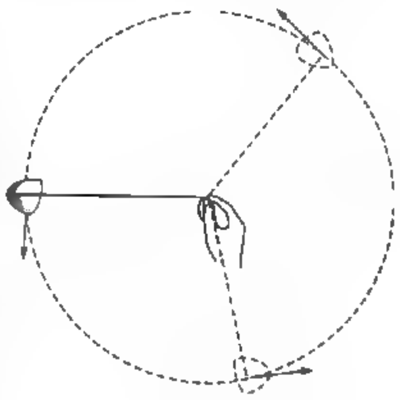


Fig. 4.2

The string tied to the stone exerts a force on it. A tight string can only pull the stone. This means that the force exerted by the string is towards the centre of the circle. If you let go of the string, it will become loose and will no longer pull the stone. The stone will fly away along a tangent to the circular path. This shows that to move an object along a circular path

with a uniform speed, a force towards the centre must be exerted on it. The force required to move an object on a circular path is called centripetal force, and it acts towards the centre of the circle.

If an object of mass m moves with a uniform speed v along a circular path of radius r, its acceleration is

$$a = \frac{v^2}{r}.$$

Then by Newton's second law of motion, the centripetal force is

$$\boxed{F = m\frac{v^2}{r}}$$

Newton's Law of Gravitation from Planetary Motion

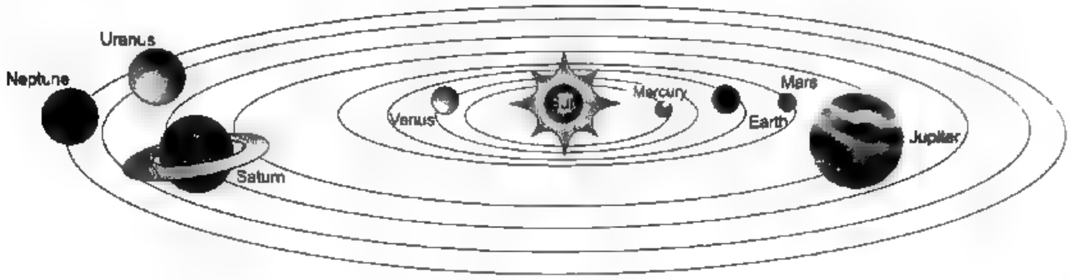


Fig. 4.3 The solar system

You know that the planets of our solar system revolve around the sun. Mercury, the planet closest to the sun, takes only 88 days to complete a revolution around the sun. The earth, the third planet from the sun, completes a revolution in 365 days. And Mars, the fourth planet from the sun, takes 687 days to complete a revolution. Kepler deduced from the observed motion of the planets that the square of a planet's time period (the time taken for one revolution) is proportional to the cube of the mean distance of the planet from the sun. This means that if planets P and Q have time periods T_1 and T_2 respectively, and they are at distances r_1 and r_2 respectively from the sun then

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}. \qquad \ldots 4.2$$

This equation is the mathematical form of Kepler's third law of planetary motion.

Consider a planet of mass m moving around the sun in a circular path (Figure 4.4). If the speed of the planet is v and its distance from the sun is r, the force acting on the planet is

$$F = m\frac{v^2}{r}. \qquad \ldots \text{(i)}$$

This force acts towards the centre of the planet's circular path, i.e., towards the sun. And it is the sun which exerts this force on the planet.

If the time period of the planet is T,

$$v = \frac{2\pi r}{T}.$$

So, in Equation (i),

$$F = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = m\frac{4\pi^2 r^2}{T^2 r} = \frac{4\pi^2 mr}{T^2}. \qquad \ldots \text{(ii)}$$

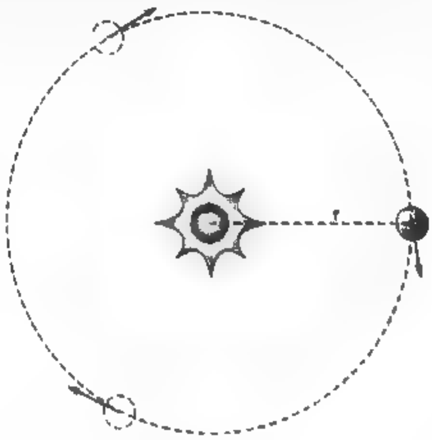


Fig. 4.4

Now, consider two planets A and B of masses m_1 and m_2 respectively. Let the forces exerted on them by the sun be F_1 and F_2 respectively. Then, from Equation (ii),

$$F_1 = \frac{4\pi^2 m_1 r_1}{T_1^2} \quad \text{and} \quad F_2 = \frac{4\pi^2 m_2 r_2}{T_2^2}.$$

So,

$$\frac{F_1}{F_2} = \frac{m_1 r_1 T_2^2}{m_2 r_2 T_1^2}.$$

Using Equation 4.2 to substitute in the above equation,

$$\frac{F_1}{F_2} = \frac{m_1 r_1}{m_2 r_2}\left(\frac{r_2^3}{r_1^3}\right)$$

or

$$\frac{F_1}{F_2} = \frac{m_1 r_2^2}{m_2 r_1^2} = \frac{m_1 / r_1^2}{m_2 / r_2^2}$$

or

$$F \propto \frac{m}{r^2}.$$

So, the force exerted on a planet by the sun is proportional to its mass as well as $1/r^2$. By Newton's third law, the force exerted on the sun by the planet has the same magnitude as the force exerted on the planet by the sun. If the force is proportional to the mass of the planet, it should be proportional to the mass of the sun (M) as well. So,

$$F \propto \frac{Mm}{r^2}$$

or

$$F = G\frac{Mm}{r^2}.$$

Newton recognized that this equation is not only true for the force between the sun and the planets, it is true for any pair of particles in the universe. The law of gravitation explains the motion of the moon, the motion of a falling stone, etc. Apart from this, it also explains phenomena such as tides, which occur due to the gravitational attraction of the moon and the sun on the waters of the oceans.

Gravitational Force Follows the Inverse Square Law

Consider two particles separated by some distance. If the distance between them doubles (increases by a factor of 2), the gravitational force between them decreases by 4 (2^2). If the distance increases by a factor of 3, the force decreases by a factor of 9, and so on. This is because gravitational force between two particles is inversely proportional to the square of the distance between them ($F \propto 1/r^2$). So, gravitational force is said to follow what is called the inverse square law. Some other kinds of forces also follow this law, i.e., their magnitudes change inversely with the square of distance.

Gravitational Forces in Some Common Cases

Consider two objects A and B placed on a horizontal table. According to the law of gravitation, each object attracts the other. In the absence of other forces, the objects should start moving towards each other. But in general, we don't see such motion. Imagine that you and your friend are sitting across a table in a restaurant. The waiter places two bowls of soup on the table. As soon as he puts the bowls down, they move towards each other because of gravitational attraction, and collide at the centre of the table. As a result, soup spills out of the bowls, spoiling your clothes. Fortunately, in real life this does not happen.

Let us estimate the magnitude of the gravitational force between the bowls of soup. Suppose each bowl has a mass of 0.25 kg. Also, suppose that they are placed at a separation of 1 m on the table. The force (approximately) on each bowl has a magnitude

$$F = G\frac{m_1 m_2}{r^2}\left[6.67\times10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}\right]\times\frac{(0.25\ \text{kg})\times(0.25\ \text{kg})}{1\ \text{m}^2} = 4.2\times10^{-12}\ \text{N}.$$

This is a very small force compared to the other forces we commonly encounter. While one bowl is pulled by the other with a force of 4.2×10^{-12} N, the table exerts a force of friction in the opposite direction, which prevents the bowl from moving.

Gravitational Force due to Earth (Gravity)

Consider the force of attraction between you (35 kg) and the earth. The mass of the earth is about 6×10^{24} kg, and its radius is about 6,400 km, i.e., 6.4×10^{6} m. The distance between you and the centre of the earth is very nearly equal to the radius of the earth (unless you are travelling in a spaceship or walking on the moon). Thus, the force of attraction on you by the earth is

$$F = G\frac{m_1 m_2}{r^2} = \left[6.67 \times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}\right]\frac{(6 \times 10^{24}\ \text{kg}) \times (35\ \text{kg})}{(6.4 \times 10^{6}\ \text{m})^2}$$

$$\approx 342\ \text{N}.$$

This force is large when compared to the force between the two soup bowls. So, we often neglect the gravitational attraction between two bodies on the earth's surface, but we do not neglect the force exerted by the earth on a body on the earth's surface.

The gravitational force exerted by the earth is often called gravity. We also talk of the sun's gravity, the moon's gravity, etc.

We used the equation $F = Gm_1m_2/r^2$ to calculate the force of attraction exerted by the earth on you. Can you justify it in view of the fact that you are not a spherical body? Because the distance r between you and the centre of the earth is so large (6,400 km), you can be treated as a point particle in this calculation. This justifies the use of this equation.

BODIES FALLING NEAR THE SURFACE OF THE EARTH

Galileo's Observations on Falling Bodies

The speed of a falling body increases as it comes down. This means that the body accelerates. Suppose you drop a coin and a feather from the same height. Which will reach the ground first? The answer is obvious, the coin will reach the ground first. Can we generalize and say that heavier objects fall faster than lighter ones? Such a generalization is not correct. If you take two solid balls of different masses, say, one of 1 kg and the other of 2 kg, and drop them from the same height, you will find that they reach the ground almost simultaneously.

It is said that Galileo dropped two stones of different masses from the Leaning Tower of Pisa (in Italy) and found that they reached the ground simultaneously. Galileo argued that the air resists an object travelling through it. If the material is dense and its surface area is small, the resistance due to air is quite small compared to the force of gravity. Thus, one can neglect the effect of air resistance while studying falling stones, metallic blocks, coins, etc. But the effect of air resistance is very important for small pieces of paper, feathers, leaves, etc., each of which has a large surface area and low density. When a coin and a feather fall through air, air offers greater resistance to the motion of the feather and less resistance to the motion of the coin. According to Galileo's argument, if air is totally removed, the coin and the feather will fall simultaneously.

Newton was born in the year Galileo died. Galileo did not have access to the equations for gravitational attraction and the acceleration resulting from a force. Still, he correctly predicted something from his observations that was contrary to everyday experience.

Galileo's prediction was tested by the British scientist Robert Boyle. He kept a coin and a feather in a long glass tube, and evacuated the air from inside the tube by using a vacuum pump. When the tube was inverted, the coin and the feather fell together.

Acceleration due to Gravity

If you drop a ball from a height, it falls. As time passes its speed increases. If you throw a ball upwards, its speed decreases till it reaches the highest point. If you throw the ball at an angle to the vertical, its direction of motion changes In all these cases, the velocity of the ball changes, i.e., the ball is accelerated. Whenever an object moves near the surface of the earth, with no other object pushing or pulling it, it is accelerated. This acceleration is caused due to the force of gravity and is called the acceleration due to gravity.

Consider an object of mass m moving freely near the earth's surface. Neglecting air resistance, the only force on it is due to gravity. The force has magnitude

$$F = \frac{GM_e m}{R_e^2}, \qquad \ldots \text{(i)}$$

where M_e = mass of the earth, m = mass of the object, and R_e = radius of the earth.

As the earth's radius R_e (6,400 km) is large as compared to distance of the object from the earth's surface, we use R_e in Equation (i) to denote the distance of the object from the centre of the earth. As the force given by Equation (i) is the resultant force on the object, its acceleration is

$$a = \frac{F}{m} = \frac{GM_e}{R_e^2}.$$

Note that this acceleration does not depend on the mass of the object. Thus, we have the following.

If gravity is the only force (meaning that air resistance is neglected), all objects move with the same acceleration near the earth's surface. This acceleration is called the acceleration due to gravity, whose magnitude 'g' is given by

$$\boxed{g = \frac{GM_e}{R_e^2}} \qquad \ldots 4.3$$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}\right)\times(6\times10^{24}\ \text{kg})}{(6.4\times10^{6}\ \text{m})^2} \approx 9.8\ \text{m/s}^2.$$

The direction of this acceleration is towards the centre of the earth, i.e., in the vertically downward direction. The acceleration has the same value, both in magnitude (9.8 m/s^2) and direction (downwards), whether the particle falls, moves up or moves at some angle with the vertical. In all these cases, we say that *the particle moves freely under gravity*.

MOTION OF BODIES MOVING FREELY UNDER GRAVITY

We have studied in Chapter 2 how to describe the motion of a particle moving along a straight line. The motion of a particle falling down or going up under gravity is also of the same kind. As the acceleration is constant (g, downward), we can use the equations

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as.$$

The position of the particle at $t = 0$ is taken as the origin. Either the downward or the upward direction is chosen as the positive direction. If the downward direction is chosen as the positive direction, $a = +g$; and if the upward direction is chosen as the positive direction, $a = -g$.

It is convenient to take the downward direction as positive when the initial velocity of an object is zero or its direction is downwards. If we take the downward direction as positive, the equations for motion under gravity can be written as

$$\boxed{\begin{aligned} v &= u + gt \\ s &= ut + \tfrac{1}{2}gt^2 \\ v^2 &= u^2 + 2gs \end{aligned}} \qquad \ldots 4.4$$

EXAMPLE 4.1 **A body is released from rest from a height. Find its speed at (a) $t = 1$ s, (b) $t = 2$ s, and (c) $t = 3$ s after the release.**

Fig. 4.5

Solution Suppose, the body is released from the point O. At 1 s, 2 s, 3 s, it is at the points A, B, C respectively (Figure 4.5).

We take vertically downwards as the positive direction.

Then $a = g = 9.8\ \text{m/s}^2$.

Here $u = 0$. Thus,

$$v = u + gt = gt.$$

(a) At $t = 1$ s,

$$v = (9.8\ \text{m/s}^2)(1\ \text{s}) = 9.8\ \text{m/s}.$$

(b) At $t = 2$ s,

$$v = (9.8\ \text{m/s}^2)(2\ \text{s}) = 19.6\ \text{m/s}.$$

(c) At $t = 3$ s,

$$v = (9.8\ \text{m/s}^2)(3\ \text{s}) = 29.4\ \text{m/s}.$$

EXAMPLE 4.2 **A particle is released from rest from a height. Find the distance it falls through in (a) 1 s, (b) 2 s, and (c) 3 s.**

Solution The situation is the same as shown in Figure 4.5.

We have $$s = ut + \frac{1}{2}gt^2.$$

As the particle is released from rest, its initial velocity is zero. Thus,

$$s = \frac{1}{2}gt^2.$$

(a) At $t = 1$ s, $$s = \frac{1}{2}(9.8\ \text{m/s}^2) \times (1\ \text{s})^2 = 4.9\ \text{m}.$$

Thus, $OA = 4.9$ m.

(b) At $t = 2$ s, $$s = \frac{1}{2}(9.8\ \text{m/s}^2) \times (2\ \text{s})^2 = 19.6\ \text{m}.$$

Thus, $OB = 19.6$ m.

(c) At $t = 3$ s, $$s = \frac{1}{2}(9.8\ \text{m/s}^2) \times (3\ \text{s})^2 = 44.1\ \text{m}.$$

Thus, $OC = 44.1$ m.

EXAMPLE 4.3 **A person takes out a coin from his pocket and drops it from a height of 1.6 m. With what speed does it strike the ground?**

Solution Here $u = 0$ and $s = 1.6$ m. We have

$$v^2 = u^2 + 2gs$$

$$= 0 + 2 \times (9.8\ \text{m/s}^2) \times (1.6\ \text{m}) = 31.36\ \text{m}^2/\text{s}^2$$

or $$v = \sqrt{31.36}\ \text{m/s} = 5.6\ \text{m/s}.$$

It is convenient to take the upward direction as positive if the direction of the initial velocity is upwards. If we take the upward direction as positive,

$$a = -g = -9.8\ \text{m/s}^2.$$

Then,

$$\begin{array}{l} v = u + gt \\ s = ut + \frac{1}{2}gt^2 \\ v^2 = u^2 + 2gs \end{array} \qquad \ldots 4.5$$

EXAMPLE 4.4 **A body is thrown upwards with a speed of 19.6 m/s. Find its speed after (a) 1 s, (b) 2 s.**

Solution Let the body be thrown from the point O. At $t = 1$ s, it reaches the point A and at $t = 2$ s, it reaches B (Figure 4.6). We take the upward direction as the positive direction. Then $u = +19.6$ m/s and $a = -g$. We have $v = u - gt$.

(a) At $t = 1$ s,

$$v = 19.6\ \text{m/s} - (9.8\ \text{m/s}^2) \times (1\ \text{s})$$
$$= 19.6\ \text{m/s} - 9.8\ \text{m/s} = 9.8\ \text{m/s}.$$

(b) At $t = 2$ s,

$$v = 19.6\ \text{m/s} - (9.8\ \text{m/s}^2) \times (2\ \text{s}) = 19.6\ \text{m/s} - 19.6\ \text{m/s} = 0.$$

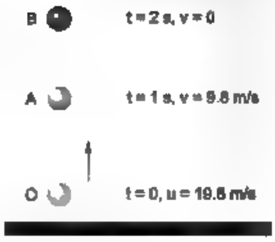


Fig. 4.6

EXAMPLE 4.5 **A body is thrown upwards with a speed of 29.4 m/s. Find the height the particle rises through in (a) 1 s, and (b) 2 s.**

Solution Taking the upward direction as positive, we have $s = ut - \frac{1}{2}gt^2$

Here $u = 29.4$ m/s.

(a) At $t = 1$ s, $s = (29.4\ \text{m/s}) \times (1\ \text{s}) - \frac{1}{2}(9.8\ \text{m/s}^2) \times (1\ \text{s})^2 = 29.4\ \text{m} - 4.9\ \text{m} = 24.5\ \text{m}.$

(b) At $t = 2$ s, $s = (29.4\ \text{m/s}) \times (2\ \text{s}) - \frac{1}{2}(9.8\ \text{m/s}^2) \times (2\ \text{s})^2 = 58.8\ \text{m} - 19.6\ \text{m} = 39.2\ \text{m}.$

So, the body moved 24.5 m in the first second and (39.2 – 24.5) m = 14.7 m in the next second. As the particle is slowing down, it moved a smaller distance in the next second.

EXAMPLE 4.6 **A particle is thrown upwards with a speed of 39.2 m/s. Find (a) the time for which it moves in the upward direction, and (b) the maximum height reached.**

Solution Here $u = 39.2$ m/s.

(a) The particle will move in the upward direction till its speed is reduced to zero. If this time is t then $v = 0$ at time t. We have

$$v = u - gt$$

or $$0 = (39.2\ \text{m/s}) - (9.8\ \text{m/s}^2)t$$

or $$t = \frac{39.2\ \text{m/s}}{9.8\ \text{m/s}^2} = 4\ \text{s}.$$

So, the particle moves upwards for 4 s.

(b) At the maximum height, $v = 0$. We have

$$v^2 = u^2 - 2gs$$

or $$0 = (39.2\ \text{m/s})^2 - 2 \times (9.8\ \text{m/s}^2)\,s$$

or $$s = \frac{39.2 \times 39.2\ \text{m}^2/\text{s}^2}{2 \times 9.8\ \text{m/s}^2} = 78.4\ \text{m}.$$

So, the maximum height reached is 78.4 m.

VARIATION IN ACCELERATION DUE TO GRAVITY WITH HEIGHT

We have calculated the acceleration due to gravity to be

$$g = \frac{GM_e}{R_e^2} = 9.8\ \text{m/s}^2.$$

This value is at the surface of the earth. If we go above the earth's surface, the value of g decreases because the distance from the earth's centre increases. At a height H above the earth's surface, the distance from the earth's centre is $R + H$. The acceleration due to gravity there will be

$$g' = G\frac{M_e}{(R_e+H)^2} = G\frac{M_e}{R_e^2}\left(\frac{R_e^2}{(R_e+H)^2}\right)$$

or
$$\boxed{g' = g\left(\frac{R_e}{R_e+H}\right)^2} \qquad \ldots 4.6$$

[illegible] **Find the value of the acceleration due to gravity at a height of 12,800 km from the surface of the earth. Earth's radius = 6,400 km.**

Solution We can use $g' = g\left(\frac{R_e}{R_e+H}\right)^2$.

Here $R_e = 6{,}400$ km and $H = 12{,}800$ km. So,

$$g' = g\left(\frac{6400\text{ km}}{6400\text{ km} + 12800\text{ km}}\right)^2 = (9.8\text{ m/s}^2)\times\left(\frac{1}{3}\right)^2 = 1.09\text{ m/s}^2.$$

G and g

The differences between G and g should be clearly understood. They have different values and different units. G is a universal constant whereas g has to be defined separately for the earth, the moon, the sun, mars, etc. In each case g will have a different value. Also, if one goes above the earth's surface or goes into a deep mine, the value of g changes.

MASS AND WEIGHT

The mass of an object is a measure of its inertia. An object's mass can be determined from the acceleration produced when a known force acts on it (from $F = ma$). The object could be anywhere—on the earth, on the moon or in a spacecraft. If you apply a given force on it, its acceleration will be the same everywhere. This means that the mass of an object remains the same everywhere. If the mass of an astronaut is 54 kg on the earth, it will remain 54 kg when he is in a spacecraft which is thousands of kilometres from the earth.

Quite often, people use the words 'mass' and 'weight' to mean the same thing. If you ask your friend what his weight is, most likely he will give his weight in kilograms. If doctors come for a free medical check-up in your school and record the weights of the students, they will do so in kilograms. But the definition of weight in physics is different.

The force with which the earth attracts an object is called the weight of the object.

Weight is a force, and hence, is expressed in newtons. Mass is expressed in kilograms. The relation between the mass m of an object and its weight W on the earth's surface is simple.

$$W = \frac{GM_e m}{R_e^2} = m\left(\frac{GM_e}{R_e^2}\right) = mg = m \times (9.8\text{ m/s}).$$

Thus, the weight of an object of mass 1 kg will be

$$W = (1\text{ kg}) \times (9.8\text{ m/s}) = 9.8\text{ N}.$$

When the reading of a weighing machine is 35 kg, your weight is 35 kg × 9.8 m/s^2 = **343 N.**

[illegible] **Find the weight of a baby of mass 5 kg.**

Solution The weight of the baby is

$$W = mg = (5\text{ kg}) \times (9.8\text{ m/s}^2) = 49.0\text{ kg m/s}^2 = 49.0\text{ N}.$$

If an object is taken to a height H above the surface of the earth, the force of attraction by the earth decreases. The acceleration due to gravity at this height is given by Equation 4.6. The weight of the object at this height is

$$W = mg'$$

or

$$\boxed{W = mg\left(\frac{R_e}{R_e + H}\right)^2} \qquad \ldots 4.7$$

[redacted] **Find the weight of an object at a height 6,400 km above the earth's surface. The weight of the object at the surface of the earth is 20 N and the radius of the earth is 6,400 km.**

Solution Here $mg = 20$ N, $R_e = 6{,}400$ km, $H = 6{,}400$ km.

Thus, $W = mg\left(\frac{R_e}{R_e + H}\right)^2 = (20\text{ N}) \times \left(\frac{6400\text{ km}}{12800\text{ km}}\right)^2 = (20\text{ N}) \times \frac{1}{4} = 5$ N.

Suppose an object is taken to such a large distance from the earth that the forces of attraction exerted by the other objects in space become important. For example, the object may be taken to the surface of the moon. Then the earth's attraction on the object will be very small compared to that of the moon.

The force by which the moon attracts an object near its surface is the weight of the object on the moon. Similarly, we can define the weight of an object at other places in the universe.

Weight of an Object on the Moon

Suppose the mass of the moon is M' and its radius is R'. Consider an object of mass m placed on its surface. The weight of the object on the moon is

$$W' = \frac{GM'm}{R'^2}$$

The weight of the same object placed on the earth's surface will be

$$W = \frac{GM_e m}{R_e^2}$$

Thus,

$$\frac{W'}{W} = \frac{M'}{M_e}\frac{R_e^2}{R'^2}. \qquad \ldots 4.8$$

Now, mass of the earth $M_e = 6 \times 10^{24}$ kg,

mass of the moon $M' = 7.4 \times 10^{22}$ kg,

radius of the earth $R_e = 6{,}400$ km, and

radius of the moon $R' = 1{,}740$ km.

So, from Equation 4.8,

$$\frac{W'}{W} = \frac{7.4 \times 10^{22}\text{ kg}}{6 \times 10^{24}\text{ kg}} \times \left(\frac{6400\text{ km}}{1740\text{ km}}\right)^2 \approx \frac{1}{6}.$$

Thus,

$$W' = \frac{W}{6}.$$

The weight of an object on the moon is about one-sixth of its weight on the earth.

Even if you do not have a calculator, you can do an approximate calculation to get this result. The mass of the earth is about 100 times that of the moon, and the radius of the earth is about 4 times that of the moon.

So, $$\frac{W'}{W} = \frac{M'}{100\,M'}\,\frac{(4R')^2}{(R')^2} = \frac{16}{100} \approx \frac{1}{6}$$

or $$W' = \frac{W}{6}.$$

EXAMPLE **Consider a heavenly body whose mass is twice that of the earth and whose radius is thrice that of the earth. What will be the weight of a book on this heavenly body, if its weight on the earth is 900 N?**

Solution The weight of the book on the earth is

$$W = \frac{GM_e m}{R_e^2}.$$

Its weight on the heavenly body is

$$W' = \frac{GM'm}{R'^2}.$$

Thus, $$\frac{W'}{W} = \frac{M'}{M_e}\,\frac{R_e^2}{R'^2} = \frac{2M_e}{M_e}\,\frac{R_e^2}{(3R_e)^2} = \frac{2}{9}$$

or $$W' = \frac{2}{9}W = \frac{2}{9} \times (900\text{ N}) = 200\text{ N}.$$

Difference between Mass and Weight

It is clear that mass and weight are two different quantities. The force with which the earth attracts a body is its weight on the earth.The mass of an object is its own property. It remains the same everywhere in the universe. But the weight of the object changes with height above the earth's surface. It is also different in different places in the universe. An object of mass 1 kg has a weight of 9.8 N at the surface of the earth, about 2.5 N at a height of 6,400 km from the earth's surface, and about 1.7 N at the surface of the moon. But, the mass of the object remains 1 kg everywhere.

Also, mass is measured in kilograms, whereas weight is measured in newtons.

SOLVED PROBLEMS

EXAMPLE **Find the gravitational force between two protons kept at a separation of 1 femtometre (1 femtometre = 10^{-15} m). The mass of a proton is 1.67×10^{-27} kg.**

Solution The gravitational force between the two protons is

$$F = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67 \times 10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}\right) \times (1.67 \times 10^{-27}\,\text{kg})^2}{(10^{-15}\,\text{m})^2}$$

$$\approx 1.86 \times 10^{-34}\text{ N}.$$

EXAMPLE **Find the gravitational force between the sun and the earth. The mass of the sun is 2.0×10^{30} kg, and the mass of the earth is 6.0×10^{24} kg. The distance between the sun and the earth is 1.5×10^{11} m.**

Solution The gravitational force between the sun and the earth is

$$F = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67 \times 10^{-11}\,\frac{\text{N m}^2}{\text{kg}^2}\right) \times (2.0 \times 10^{30}\,\text{kg}) \times (6.0 \times 10^{24}\,\text{kg})}{(1.5 \times 10^{11}\,\text{m})^2}$$

$$\approx 3.56 \times 10^{22}\text{ N}.$$

EXAMPLE **A particle is taken to a height of $2R_e$ above the earth's surface, where R_e is the radius of the earth. If it is dropped from this height, what would be its acceleration?**

Solution The acceleration due to gravity at height H above the surface of the earth is

$$g' = g\left(\frac{R_e}{R_e + H}\right)^2.$$

Here $H = 2R_e$. Thus,

$$g' = g\left(\frac{R_e}{3R_e}\right)^2 = \frac{g}{9} = \frac{9.8\ \text{m/s}^2}{9} \approx 1.1\ \text{m/s}^2.$$

EXAMPLE **Consider a heavenly body whose mass is 3×10^{24} kg (half that of the earth) and radius is 3,200 km (half that of the earth). What is the acceleration due to gravity at the surface of this heavenly body?**

Solution

$$g = \frac{GM}{R^2} = \frac{G(M_e/2)}{(R_e/2)^2},$$

where M_e and R_e are the mass and the radius of the earth respectively. Thus,

$$g = 2\frac{GM_e}{R_e^2} = 2 \times 9.8\ \text{m/s}^2 = 19.6\ \text{m/s}^2.$$

EXAMPLE **Two bodies A and B of masses m and $2m$ respectively are kept a distance d apart. Where should a small particle be placed, so that the net gravitational force on it due to the bodies A and B is zero?**

Solution It is clear that the particle must be placed on the line AB between the bodies A and B. Suppose it is at a distance x from A. Let its mass be m'

The force on m' due to A is

$$F_1 = \frac{Gmm'}{x^2} \text{ towards A}$$

and that due to B is

$$F_2 = \frac{G(2m)m'}{(d-x)^2} \text{ towards B.}$$

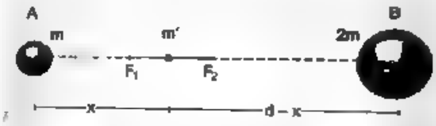


Fig. 4.W1

The net force will be zero if $F_1 = F_2$.

Thus, $$\frac{Gmm'}{x^2} = \frac{G(2m)m'}{(d-x)^2}$$

or $$(d-x)^2 = 2x^2$$

or $$d - x = \pm\sqrt{2}\,x$$

or $$d = (1 \pm \sqrt{2})\,x.$$

Thus, $$x = \frac{d}{1+\sqrt{2}} \quad \text{or} \quad \frac{d}{1-\sqrt{2}}.$$

As x cannot be negative,

$$x = \frac{d}{1+\sqrt{2}}.$$

EXAMPLE **Two bodies of masses 1 kg and 2 kg respectively are placed at a separation of 1 m. Find the accelerations of the bodies assuming that only gravitational forces act.**

Solution The force of gravitation is

$$F = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}\right) \times (1\ \text{kg}) \times (2\ \text{kg})}{(1\ \text{m})^2}$$

$$= 1.33 \times 10^{-10}\ \text{N}.$$

Each body attracts the other with a force of this magnitude. The acceleration of the 1-kg body is

$$a = \frac{F}{1\text{ kg}} = \frac{1.33\times 10^{-10}\text{ N}}{1\text{ kg}} = 1.33\times 10^{-10}\text{ m/s}^2.$$

The acceleration of the 2-kg body is

$$a = \frac{F}{2\text{ kg}} = 6.67\times 10^{-11}\text{ m/s}^2.$$

EXAMPLE 7 **A ball is thrown upwards with some initial speed. It goes up to a height of 19.6 m and then returns. Find (a) the initial speed, (b) the time taken in reaching the highest point, (c) the velocity of the ball one second before and one second after it reaches the maximum height, and (d) the time taken by the ball to return to its original position.**

Solution (a) At the maximum height, the velocity becomes zero. Taking the upward direction as positive,

$$v^2 = u^2 - 2gs$$

or $$0 = u^2 - 2\times(9.8\text{ m/s}^2)\times(19.6\text{ m})$$

or $$u^2 = (2\times 9.8\times 19.6)\text{ m}^2/\text{s}^2$$

or $$u = 19.6\text{ m/s}.$$

(b) We have $$v = u - gt$$

or $$0 = \left(19.6\,\frac{\text{m}}{\text{s}}\right) - (9.8\text{ m/s}^2)\times t$$

or $$t = \frac{19.6\text{ m/s}}{9.8\text{ m/s}^2} = 2\text{ s}.$$

So the particle takes 2 s to reach the highest point.

(c) The ball reaches the maximum height at $t = 2$ s. We have to find the velocity at $t = 1$ s and $t = 3$ s.

We have $$v = u - gt.$$

At $t = 1$ s,

$$v = \left(19.6\,\frac{\text{m}}{\text{s}}\right) - (9.8\text{ m/s}^2)\times(1\text{ s}) = 19.6\,\frac{\text{m}}{\text{s}} - 9.8\,\frac{\text{m}}{\text{s}} = 9.8\text{ m/s}.$$

As v is positive, the particle is going up with a speed of 9.8 m/s.

At $t = 3$ s,

$$v = u - gt = \left(19.6\,\frac{\text{m}}{\text{s}}\right) - (9.8\text{ m/s}^2)\times(3\text{ s}) = 19.6\,\frac{\text{m}}{\text{s}} - 29.4\,\frac{\text{m}}{\text{s}} = -9.8\text{ m/s}.$$

As v is negative, the particle is coming down with a speed of 9.8 m/s.

(d) When the ball returns to its original position, its displacement $s = 0$.

Using $$s = ut - \frac{1}{2}gt^2,$$

$$0 = \left(19.6\,\frac{\text{m}}{\text{s}}\right)t - \frac{1}{2}\times(9.8\text{ m/s}^2)\,t^2.$$

As $t \neq 0$, $$19.6\,\frac{\text{m}}{\text{s}} = (4.9\text{ m/s}^2)\times t$$

or $$t = \frac{19.6\text{ m/s}}{4.9\text{ m/s}^2} = 4\text{ s}.$$

So, the ball takes 4 s to return to its original position.

EXAMPLE 8 **A body is dropped from some height. It moves through a distance of 24.5 m in the last second before hitting the ground. Find the height from which it was dropped.**

Solution Suppose the particle is dropped from a height of x metres. The total time taken by it in falling is t seconds, where

$$x = \frac{1}{2}(9.8)t^2 = 4.9\,t^2. \qquad \ldots(1)$$

The distance moved in $(t-1)$ s is x_1 metres, where

$$x_1 = 4.9\,(t-1)^2. \qquad \ldots\text{(ii)}$$

The distance moved in the last second is $(x - x_1)$ metres. From (i) and (ii),

$$x - x_1 = 4.9\,t^2 - 4.9(t-1)^2 = 4.9\,(t^2 - t^2 + 2t - 1)$$

or $$24.5 = 9.8\,t - 4.9 \quad \text{or} \quad 9.8\,t = 24.5 + 4.9 = 29.4$$

or $$t = \frac{29.4}{9.8} = 3.$$

From (i),

$$x = 4.9\,t^2 = 4.9 \times 9 = 44.1.$$

So, the particle was dropped from a height of 44.1 metres.

[redacted] **A ball is dropped from the edge of a roof. It takes 0.1 s to cross a window of height 2.0 m. Find the height of the roof above the top of the window.**

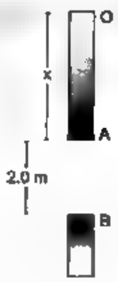


Fig. 4.W2

Solution Let AB be the window, and suppose the roof is at a height x above A. Also, suppose it takes a time t_1 for the ball to reach A.
The velocity of the ball at A is

$$v_1 = 0 + gt_1 = (9.8\ \text{m/s}^2)\,t_1.$$

Now, consider the motion of the ball from A to B. Here the initial velocity is v_1, the distance covered is 2 m, and the time taken is 0.1 s. We have

$$s = ut + \frac{1}{2}gt^2$$

or $$2.0\ \text{m} = v_1\,(0.1\ \text{s}) + \frac{1}{2} \times (9.8\ \text{m/s}^2) \times (0.1\ \text{s})^2 = (9.8\ \text{m/s}^2)\,t_1\,(0.1\ \text{s}) + 0.049\ \text{m}$$

or $$2.0\ \text{m} - 0.049\ \text{m} = \left(0.98\,\frac{\text{m}}{\text{s}}\right)t_1$$

or $$t_1 = \frac{1.951\ \text{m}}{0.98\ \text{m/s}} \approx 2\ \text{s}.$$

The height x is

$$x = \frac{1}{2}gt_1^2 = \frac{1}{2} \times (9.8\ \text{m/s}^2) \times (2\ \text{s})^2 = 19.6\ \text{m}$$

The roof is at a height 19.6 m above the top of the window.

[redacted] **A ball is dropped from a height of 20 m. At the same instant another ball is thrown up from the ground with a speed of 20 m/s. When and where will the balls meet?**

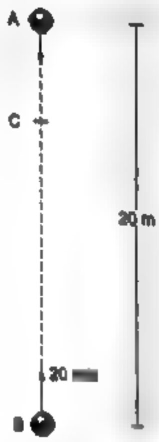


Fig. 4.W3

Solution Suppose the first ball is dropped from the point A and the second is thrown up from the point B as shown in Figure 4.W3. Suppose the balls meet at C at time t. The ball A starts from rest. So,

$$AC = \frac{1}{2}gt^2. \qquad \ldots\text{(i)}$$

At B, the ball is thrown up with a speed of 20 m/s. The displacement in time t is

$$BC = (20\ \text{m/s})\,t - \frac{1}{2}gt^2. \qquad \ldots\text{(ii)}$$

Adding (i) and (ii),

$$AC + BC = (20\ \text{m/s})\,t$$

or $$20\ \text{m} = (20\ \text{m/s})\,t$$

or $$t = \frac{20\ \text{m}}{20\ \text{m/s}} = 1\ \text{s}.$$

So, the balls meet at $t = 1$ s after they start moving.

From (ii),

$$BC = (20\ \text{m/s})\,(1\ \text{s}) - \frac{1}{2}(9.8\ \text{m/s}^2) \times (1\ \text{s})^2 = 20\ \text{m} - 4.9\ \text{m} = 15.1\ \text{m}.$$

So, the balls meet at a height 15.1 m above the ground.

EXAMPLE **A ball A is dropped from a 44.1-m-high cliff. Two seconds later, another ball B is thrown downwards from the same place with some initial speed. The two balls reach the ground together. Find the speed with which the ball B was thrown.**

Solution The time taken by the ball A to reach the ground is given by

$$44.1\ \text{m} = \frac{1}{2} \times (9.8\ \text{m/s}^2) \times t^2$$

or $$t^2 = \frac{2 \times 44.1\ \text{s}^2}{9.8} = 9\ \text{s}^2$$

or $$t = 3\ \text{s}.$$

Suppose the ball B was thrown with an initial speed u. It was thrown 2 s after A was dropped and it reached the ground together with A. So, the total time for which it has fallen is (3 − 2) s = 1 s. In this time the ball B has covered a distance of 44.1 m. We have,

$$s = ut + \frac{1}{2}gt^2$$

or $$44.1\ \text{m} = u\,(1\ \text{s}) + \frac{1}{2}(9.8\ \text{m/s}^2) \times (1\ \text{s})^2$$

or $$u\,(1\ \text{s}) = 44.1\ \text{m} - 4.9\ \text{m} = 39.2\ \text{m}$$

or $$u = \frac{39.2\ \text{m}}{1\ \text{s}} = 39.2\ \text{m/s}.$$

EXAMPLE **Suppose an astronaut lands on the moon and drops an object from a height of 7.35 m from the surface. How much time will it take to reach the moon's surface?**

Solution The acceleration of the object due to the moon's gravity will be $a = g/6 = \frac{9.8}{6}\ \text{m/s}^2$. If the object takes a time t to reach the surface,

$$s = \frac{1}{2}at^2$$

or $$7.35\ \text{m} = \frac{1}{2} \times \frac{9.8}{6}\ \text{m/s}^2 \times t^2$$

or $$t^2 = \frac{2 \times 7.35 \times 6}{9.8}\ \text{s}^2 = 9\ \text{s}^2$$

or $$t = 3\ \text{s}.$$

The object will take 3 s to reach the moon's surface.

EXAMPLE **Communication satellites move in orbits of radius 44,400 km around the earth. Find the acceleration of such a satellite, assuming that the only force acting on it is that due to the earth. Mass of the earth = 6×10^{24} kg.**

Solution The force on the satellite due to the earth is

$$F = \frac{GM_e M_{sat}}{d^2},$$

where d = 44,400 km is the distance of the satellite from the earth's centre.

Acceleration $$a = \frac{F}{M_{sat}} = \frac{GM_e}{d^2} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(44400 \times 10^3)^2}\ \text{m/s}^2$$

$$\approx 0.2\ \text{m/s}^2.$$

• POINTS TO REMEMBER •

- *Universal law of gravitation*

  The gravitational force of attraction between any two particles is directly proportional to the product of the masses of the particles and is inversely proportional to the square of the distance between the particles. The direction of the force is along the line joining the two particles.

- *Universal constant of gravitation*

  The constant G appearing in Newton's law of gravitation is called the 'universal constant of gravitation'. Its value is

  $$6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}.$$

- The gravitational force due to a heavenly body such as the earth is called gravity.
- If gravity is the only force, all bodies near the earth's surface move with the same acceleration called 'acceleration due to gravity'.
- *Acceleration due to gravity*

  The acceleration due to gravity near the earth's surface is $g = \frac{GM_e}{R_e^2} = 9.8 \text{ m/s}^2$.

  The direction of this acceleration is vertically downwards, that is, towards the earth's centre.

  The acceleration of a body moving near the earth's surface under the influence of gravity is the same regardless of whether it moves up, falls down or moves at some angle to the vertical.

- If the downward direction is taken as the positive direction, the acceleration of a particle moving near the earth's surface is $a = +g$. If the upward direction is taken as the positive direction, the acceleration is $a = -g$.
- *Variation in acceleration due to gravity with height*

  As one goes above the earth's surface, the value of acceleration due to gravity decreases. At a height H above the surface, the value is

  $$g' = g\left[\frac{R_e}{R_e + H}\right]^2.$$

  If one goes deep inside the earth then also the value of g decreases.

- *Weight and mass*

  The force with which the earth attracts a body is called the weight of the body on the earth. The weight changes with height above the earth's surface, but the mass remains the same everywhere.

  The weight of a body on the moon is about one-sixth of its weight on the earth.

• EXERCISES •

A. ***Objective Questions***

I. *Pick the correct option.*

1. Two particles are kept at a separation r. The gravitational force between them is proportional to
   (a) r (b) r^2 (c) $1/r$ (d) $1/r^2$

2. The universal constant of gravitation G has the unit
   (a) N (b) m/s
   (c) $(\text{N m}^2)/\text{kg}^2$ (d) J

3. The equation $F = \frac{Gm_1m_2}{r^2}$ is valid for
   (a) rectangular bodies (b) circular bodies
   (c) elliptical bodies (d) spherical bodies

4. The force acting on a ball due to the earth has a magnitude F_b and that acting on the earth due to the ball has a magnitude F_e. Then,
   (a) $F_b = F_e$ (b) $F_b > F_e$
   (c) $F_b < F_e$ (d) $F_e = 0$

5. The force of gravitation between two bodies of mass 1 kg each kept at a distance of 1 m is
   (a) 6.67 N (b) 6.67×10^{-9} N
   (c) 6.67×10^{-7} N (d) 6.67×10^{-11} N

6. The earth attracts a body of mass 1 kg kept on its surface with a force of
   (a) 1 N (b) 6.67×10^{-11} N
   (c) 9.8 N (d) $\frac{1}{9.8}$ N

7. A coin and a feather are dropped together in a vacuum.
   (a) The coin will reach the ground first.
   (b) The feather will reach the ground first.
   (c) Both the bodies will reach the ground together.
   (d) The feather will not fall down.

8. Two bodies A and B of masses 100 g and 200 g respectively are dropped near the earth's surface. Let the accelerations of A and B be a_1 and a_2 respectively.
   (a) $a_1 = a_2$ (b) $a_1 < a_2$ (c) $a_1 > a_2$ (d) $a_1 \neq a_2$

9. Newton's law of gravitation is valid
   (a) on the earth only
   (b) on the moon only
   (c) in the laboratory only
   (d) everywhere

10. The acceleration due to gravity is 9.8 m/s^2
    (a) much above the earth's surface
    (b) near the earth's surface

(c) deep inside the earth
(d) at the centre of the earth

11. The acceleration due to gravity near the moon's surface is
    (a) approximately equal to that near the earth's surface
    (b) approximately six times that near the earth's surface
    (c) approximately one-sixth of that near the earth's surface
    (d) slightly greater than that near the earth's surface

12. A particle is taken to a height R above the earth's surface, where R is the radius of the earth. The acceleration due to gravity there is
    (a) $2.45\ m/s^2$ (b) $4.9\ m/s^2$
    (c) $9.8\ m/s^2$ (d) $19.6\ m/s^2$

13. Consider a heavenly body that has mass $2M_e$ and radius $2R_e$, where M_e and R_e are the mass and the radius of the earth respectively. The acceleration due to gravity at the surface of this heavenly body is
    (a) $2.45\ m/s^2$ (b) $4.9\ m/s^2$
    (c) $9.8\ m/s^2$ (d) $19.6\ m/s^2$

14. When a body is thrown up, the force of gravity is
    (a) in the upward direction
    (b) in the downward direction
    (c) zero
    (d) in the horizontal direction

15. The mass of a body is measured to be 12 kg on the earth. If it is taken to the moon, its mass will be
    (a) 12 kg (b) 6 kg (c) 2 kg (d) 72 kg

16. The weight of a body is 120 N on the earth. If it is taken to the moon, its weight will be about
    (a) 120 N (b) 60 N (c) 20 N (d) 720 N

II. *Mark the statements true (T) or false (F).*

1. Newton's law of gravitation follows from Newton's laws of motion.
2. The force of gravitation exerted by the earth on a ball is Gm_1m_2/r^2, where r is the distance of the ball from the earth's surface.
3. Due to gravitational forces, all bodies in the universe attract each other.
4. The force of gravitation between two particles is along the line joining the two particles.
5. The value of G cannot be measured in a laboratory.
6. G and g are two ways of writing the same quantity.
7. The value of G on the moon is about one-sixth of its value on the earth.
8. The value of g at the centre of the earth is zero.
9. Mass and weight are measured in different units.
10. Mass decreases with height above the earth's surface

III. *Fill in the blanks.*

1. The force of gravitation exerted on one body by the other is F. If the mass of each body is doubled, the force will become ....
2. The force of gravitation between two objects is F when they are kept at some separation on the earth's surface. If the objects are kept at the same separation on the moon's surface, the force of gravitation between them will be ....
3. If the distance between two objects is doubled, the force of gravitational attraction between them becomes ...... of its initial value.
4. The force of gravitation between two spherical bodies is Gm_1m_2/r^2, where r is the separation between their ....
5. If the earth shrinks, thus decreasing its radius, the value of g at its surface will ....
6. The unit of weight is ....

B. *Very-Short-Answer Questions*

Answer the following in one word or maximum one sentence.

1. You know that the earth attracts you in the vertically downward direction. Do you attract the earth as well? If yes, in which direction?
2. Write the units of G and g.
3. Consider two bodies A and B. The body B is heavier than A. Which of them is attracted with a greater force by the earth? Which will fall with a greater acceleration?
4. A coin and a feather are dropped from the roof of a building. Which will fall to the ground first?
5. What do you mean by the weight of a body on the moon?
6. The weight of a body on the moon is about one-sixth of its weight on the earth. Is the mass of the body on the moon also one-sixth of its mass on the earth?

C. *Short-Answer Questions*

Answer the following in about 30–40 words.

1. Why is Newton's law of gravitation called a universal law?
2. Why do we neglect gravitational forces exerted on a falling stone by nearby trees while calculating its acceleration?
3. A piece of paper takes much longer to fall than a stone through the same distance. Explain the reason.
4. Starting from Newton's law of gravitation, show that the value of g decreases as one goes above the earth's surface.
5. What do you mean by the weight of a body? What is its unit?
6. An 80-kg patient went to a doctor. The doctor advised him to reduce his weight. Should the patient plan to live on the moon so that his weight gets reduced? Explain your answer.

D. *Long-Answer Questions*

Answer the following in not more than 70 words.

1. If each object in the universe attracts every other object, why don't two books kept on a table come towards each other and collide?
2. State Newton's law of gravitation and write the mathematical equation describing it.
3. What are the differences between G and g? Are both of them universal constants?
4. Show from Newton's law of gravitation and Newton's second law of motion that the acceleration of a freely falling body does not depend on the mass of the body.
5. What is the relation between the mass m and the weight W of a body? What are the differences between the two?
6. The acceleration due to gravity for falling bodies is taken to be constant. Justify this in view of the fact that the distance of a falling body from the earth's surface keeps changing.
7. Show that if a body is taken to a height H above the earth's surface, acceleration due to gravity is decreased by the factor $R^2/(R+H)^2$, where R is the radius of the earth.
8. Show that the acceleration due to gravity at the surface of the moon is about one-sixth of that at the surface of the earth.

E. *Numerical Problems*

1. Calculate the gravitational force between a 10-kg ball and a 20-kg ball placed at a separation of 5 m.
2. Three balls A, B and C are kept in a straight line. The separation between A and C is 1 m, and B is placed at the midpoint between them. The masses of A, B, C are 100 g, 200 g and 300 g respectively. Find the net gravitational force on (a) A, (b) B, and (c) C.
3. A particle of mass m_1 is kept at $x = 0$ and another of mass m_2 at $x = d$. When a third particle is kept at $x = d/4$, it experiences no net gravitational force due to the two particles. Find m_2/m_1.
4. The acceleration due to gravity near the earth's surface is 9.8 m/s^2, and the earth's radius is 6,400 km. From this data calculate the mass of the earth. Use any universal constant if required.
5. Two particles of mass 200 g each are placed at a separation of 10 cm. Assume that the only forces acting on them are due to their gravitational attraction. Find the acceleration of each when they are allowed to move.
6. A particle weighs 120 N on the surface of the earth. At what height above the earth's surface will its weight be 30 N? Radius of the earth = 6,400 km.
7. Suppose the earth shrinks such that its radius decreases to half the present value. What will be the acceleration due to gravity on the surface of the earth?
8. A body weighs 120 N on the earth. Find its approximate weight on the moon.
9. Calculate the value of the acceleration due to gravity at a place 3,200 km above the surface of the earth.
10. The acceleration due to gravity at a place is 0.2 m/s^2. Find its height above the earth's surface.
11. As one moves to a place 3,200 km above the earth's surface, the acceleration due to gravity reduces to 4/9 of its value at the earth's surface. Calculate the radius of the earth from this data.
12. A ball is dropped from a cliff. Find (a) its speed 2 s after it is dropped, (b) its speed when it has fallen through 78.4 m, and (c) the time taken in falling through 78.4 m.
13. A ball is thrown upwards with a speed of 39.2 m/s. Calculate (a) the maximum height it reaches, and (b) the time taken in reaching the maximum height.
14. A ball thrown upwards takes 4 s to reach the maximum height. Find (a) the initial speed with which it was thrown, and (b) the maximum height reached.
15. An object thrown upwards reaches the highest point in 5.0 s. Find the velocity with which it was thrown.
16. A stone thrown upwards attains a maximum height of 19.6 m. Find the velocity with which it was thrown.
17. A body is thrown upwards with a velocity of 20 m/s. How much time will it take to return to its original position?
18. A ball is dropped from a height 2.50 m above the floor. (a) Find the speed v with which it reaches the floor. (b) The ball now rebounds. The speed of the ball is decreased to $3v/4$ due to this collision. How high will the ball rise?
19. A stone is dropped from a cliff at 2:30:30 p.m. (hour:minute:second). Another stone is dropped from the same point at 2:30:31 p.m. Find the separation between the stones at (a) 2:30:31 p.m., (b) 2:30:35 p.m.
20. A ball is thrown upwards from the surface of the moon with a velocity of 19.6 m/s. (a) How much time will it take to attain the maximum height? (b) How high will it go?
21. A flowerpot drops from the edge of the roof of a multistoried building. Calculate the time taken by the pot to cross a particular distance AB of height 2.9 m, the upper point A being 19.6 m below the roof.
22. A wicketkeeping glove is dropped from a height of 40 m and simultaneously a ball is thrown upwards from the ground with a speed of 40 m/s. When and where do they meet?
23. A boy on a 78.4-m-high cliff drops a stone. One second later, he throws another stone downwards with some speed. The two stones reach the ground simultaneously. Find the speed with which the second stone was thrown.

• ANSWERS •

A. *Objective Questions*

I. **1.** (d) **2.** (c) **3.** (d) **4.** (a) **5.** (d)
6. (c) **7.** (c) **8.** (a) **9.** (d) **10.** (b)
11. (c) **12.** (a) **13.** (b) **14.** (b) **15.** (a)
16. (c)

II. **1.** F **2.** F **3.** T **4.** T **5.** F
6. F **7.** F **8.** T **9.** T **10.** F

E. *Numerical Problems*

1. 5.34×10^{-10} N
2. (a) 7.34×10^{-12} N towards C
(b) 1.07×10^{-11} N towards C
(c) 1.80×10^{-11} N towards A
3. 9 **4.** 6.02×10^{24} kg
5. 1.33×10^{-9} m/s^2 **6.** 6,400 km
7. 39.2 m/s^2 **8.** 20 N **9.** 4.36 m/s^2
10. $6R_e = 38{,}400$ km **11.** 6,400 km
12. (a) 19.6 m/s (b) 39.2 m/s (c) 4 s
13. (a) 78.4 m (b) 4 s **14.** (a) 39.2 m/s (b) 78.4 m
15. 49 m/s **16.** 19.6 m/s **17.** 4.08 s
18. (a) 7 m/s (b) 1.4 m **19.** (a) 4.9 m (b) 44.1 m
20. (a) 12 s (b) 117.6 m **21.** $\frac{1}{7}$ s
22. 1 s after the glove is dropped, 35.1 m above the ground **23.** 11.43 m/s

• POSTSCRIPT •

• *Kepler's laws*

The Danish astronomer Tycho Brahe spent more than 20 years measuring the positions of the sun and the planets at different times. Brahe took Johannes Kepler as his student during the final years of his work. Kepler had a wonderful ability in mathematical computations. Kepler analysed the data collected by Brahe and put forward three laws of planetary motion.

1. All planets move in elliptical orbits with the sun at the focus.

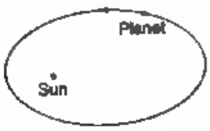


Fig. 4.A1

2. The line joining a planet to the sun sweeps equal areas in equal times.
3. The square of the time period (time taken to complete a revolution) is proportional to the cube of the average distance of the planet from the sun.

• *Sirius and its companion*

There are a number of stars which one can see with naked eyes. There are many more which can be seen through telescopes. There are certain pairs of stars which are close to each other and are, therefore, bound to each other due to their own gravitational forces. Each moves round the other in an elliptical path. As seen from the earth the pair moves together. A pair of such stars are called *binary stars*.

One such pair was detected by an astronomer called Friedrich Wilhelm Bessel in 1844. One of the two stars in the pair is named Sirius, and it is the brightest star seen at night. It is also called the Dog Star. It has mass 2.2 times that of the sun and emits about 23 times more radiation than the sun. It does not appear 23 times brighter than the sun because it is much farther away. The other star in the pair is called the companion of Sirius, and it has a mass 0.9 times that of the sun. It is also called the *pup* of Dog Star. This star is so small that it can be seen only through very powerful telescopes. In fact, Bessel could not see it in 1844 when he predicted its existence. Because of its presence, the path of Sirius got curved and from this observation, Bessel predicted that Sirius has a companion. This companion was actually observed in 1864 when larger telescopes were built.

5 Work, Energy and Power

WORK

The word 'work' has many different meanings. I work in a factory The Ramayana is a great work of Valmiki. The machine is in working order. There are a large number of worked-out problems in this book. These are just some ways in which we use the word 'work'. In physics, however, it has a special meaning. It represents a physical quantity defined in a special way which we shall describe below.

When a force acts on an object and the object moves, we say that the force has done work on the object.

When an apple falls from a tree, the force of attraction of the earth does work on the apple. When you throw a ball, you exert a force on the ball as long as it is in your hand. The ball moves, and we say that the force exerted by the hand has done work on the ball. When you push a book on a table, you exert a force on the book. As the book moves, this force does work.

Work is always done by a force. We often name the agent that has applied this force and say that the agent has done work. For example, we say that the earth has done work on the apple. In fact, work is done by the force of attraction that the earth has exerted on the apple. Similarly, when a girl pushes a book on the table, we say that the girl has done work on the book. Actually, the work is done by the force exerted by the girl.

The work done by a force acting on an object is equal to the product of the force and the displacement of the object in the direction of the force.

We assume here that the force remains constant during the whole displacement.

Note that both force and displacement have directions, i.e., they are vectors. Work, which is a product of force and displacement, does not have a direction. It is a scalar quantity.

Unit of Work

Work done is the product of force and displacement. Using the SI units of force and displacement, i.e., newton and metre respectively, the unit of work comes out to be 'newton metre'. This is given a separate name, joule, in honour of the British scientist James Prescott Joule. The symbol of joule is J.

When a force of 1 N acts on an object and the object moves a distance of 1 m in the direction of the force, the work done by the force is 1 J.

Calculation of Work

The direction of the displacement of an object can have different relations with the direction of the force acting on it. Their directions may be the same, opposite, perpendicular to each other, at an angle, etc. Let us see how work is calculated in these situations.

Displacement in the direction of the force

If the displacement of an object is in the direction of the force applied on it, the amount of the work done by the force on this object is obtained by multiplying the force and the displacement.

➤ **When Displacement is Along the Force** *work done = force × displacement*

If we denote work, force and displacement by W, F and d respectively then

$$W = Fd. \qquad \text{...5.1}$$

For example, when an object falls, its displacement is in the direction of the force of gravity. Similarly, if you push a book along a table, the displacement of the book is along the direction of the force you exert. We can use Equation 5.1 in these cases.

EXAMPLE 5.1 **A boy pushes a book by applying a force of 5.0 N. Find the work done by this force in displacing the book through 20 cm along the direction of the push.**

Solution Work done is

$$\begin{aligned} W &= Fd \\ &= (5.0\text{ N}) \times (20\text{ cm}) \\ &= (5.0\text{ N}) \times (0.2\text{ m}) \\ &= 1.0\text{ N m} = 1.0\text{ J} \end{aligned}$$

Displacement in the direction opposite to the force

In certain cases, the displacement of an object may be in the direction opposite to the direction of the force acting on it. If a ball is thrown up, its displacement is in the upward direction, whereas, the force due to the earth's gravity is in the downward direction. We can say that the object has moved a negative distance in the vertically downward direction.

If the force acting on an object and its displacement are in opposite directions, the work done by the force on the object is

$$W = -Fd, \qquad \text{...5.2}$$

where F is the force and d is the displacement of the object.

➤ **When Displacement is Opposite the Force** *work done = – (force × displacement)*

EXAMPLE 5.2 **A ball of mass 1 kg thrown upwards reaches a maximum height of 5.0 m. Calculate the work done by the force of gravity during this vertical displacement.**

Solution The force of gravity on the ball is

$$F = mg = (1\text{ kg}) \times (9.8\text{ m/s}^2) = 9.8\text{ N}.$$

The displacement of the ball is $d = 5.0$ m.

The force and the displacement are in opposite directions. Hence,

$$\begin{aligned} W &= -Fd \\ &= -(9.8\text{ N}) \times (5.0\text{ m}) = -49\text{ J}. \end{aligned}$$

We can also say that a work of 49 J has been done *against* the force of gravity.

Displacement in the direction perpendicular to the force

If the displacement of an object is perpendicular to the force acting on it, the work done by the force on the object is zero. Consider, for example, the force of gravity acting on an aeroplane flying in the sky. The force of gravity is in the downward direction, whereas the aeroplane's displacement is in the horizontal direction, i.e., the force and the displacement are perpendicular to each other. There

Fig. 5.1

is no displacement in the direction of the force of gravity, and therefore, the work done by it on the aeroplane is zero.

When Displacement is Perpendicular to the Force *work done = zero*

When a porter moves on a railway platform with a heavy load on his head, he exerts a vertically upward force on the load. But, the displacement of the load is in the horizontal direction. The load has not moved any distance in the vertical direction, and hence, the work done by the force exerted by the porter is zero. The porter does no work on the load when he moves on the railway platform. (Why do people pay him!)

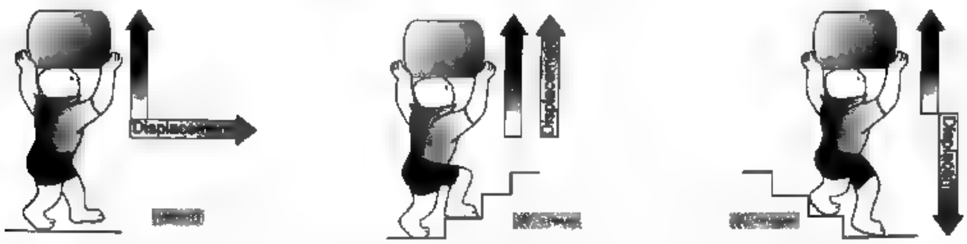


Fig. 5.2 Work done (W) by a porter on the load in different situations. The arrows indicate the directions of the displacement and the force exerted by the porter on the load.

Does the porter do any work when he climbs up a flight of stairs or comes down the stairs? When he climbs up the stairs, he does positive work on the load as there is upward displacement. When he comes down the stairs, he does negative work on the load. This is because the load is displaced in the vertically downward direction, whereas the force exerted by the porter on the load is in the upward direction.

Displacement at an angle to the force

Consider an object of mass m sliding on an inclined surface (Figure 5.3). The object moves along the inclined surface from A to B. Its displacement during this period is AB. The force of gravity on this object is mg, which acts in the vertically downward direction. What is the work done by the force mg on the object as it moves from A to B?

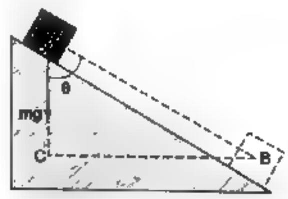


Fig. 5.3

As the object moves from A to B, it comes down through a height AC. We can say that the object is displaced in the vertically downward direction through a distance AC. (At the same time, it is also displaced horizontally through a distance CB.) Since the displacement *in the direction of the force* is AC, the work done is

$$W = mg\,(AC)$$
$$= mg\,(AB\cos\theta). \qquad \ldots(i)$$

Fig. 5.4

In general, if the displacement d of an object makes an angle θ with the force F acting on it (Figure 5.4), the work done by the force is

$$\boxed{W = Fd\cos\theta} \qquad \ldots 5.3$$

In Figure 5.3, AB is the actual displacement. $AB\cos\theta$ is the displacement in the direction of AC. We say that $AB\cos\theta$ is *the component of the displacement* (AB) *along* AC.

To find the component of displacement in a given direction, simply multiply the magnitude of the displacement by $\cos\theta$, where θ is the angle between the actual displacement and the direction in which you need the component. (The values of $\cos\theta$ for some angles are given at the bottom of this page.)

We can also use this method to get the component of a force in a given direction. In Figure 5.3, mg, the force of gravity on the block, acts vertically downwards. The line AB makes an angle θ with the direction of mg. Thus, the component of the force mg along AB is $mg\cos\theta$.

In Figure 5.4, $d\cos\theta$ is the component of the displacement d along the direction of the force F. Also, $F\cos\theta$ is the component of the force F along the displacement. Thus, Equation 5.3 can be written in the two ways given below.

General Equation for Work Done

work = force × component of the displacement along the force

or

work = displacement × component of the force along the displacement

Equation 5.3 is the general definition of work. If the displacement is along the force, $\theta = 0$, $\cos\theta = 1$. So, $W = Fd$ as in Equation 5.1. If the displacement is opposite to the force, $\theta = 180°$, $\cos\theta = -1$. So, $W = -Fd$ as in Equation 5.2. If the displacement is perpendicular to the force, $\theta = 90°$, $\cos\theta = 0$. So, $W = Fd\cos 90° = 0$.

EXAMPLE 5.3 **A person pulls a block on a horizontal surface by applying a force of 5.0 N at an angle of 30° with the horizontal (Figure 5.5). Find the work done by this force in displacing the block through 2.0 m.**

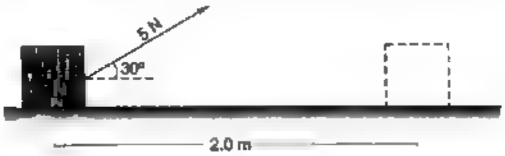


Fig. 5.5

Solution The work done by the force is

$$W = Fd\cos\theta$$
$$= (5.0\text{ N})\times(2.0\text{ m})\cos 30° = (5.0\text{ N})\times(2.0\text{ m})\frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ J}.$$

ENERGY

The word 'energy' is used in many contexts, just like the word 'work' is. We hear people talk of a person's energy, energy-giving tonics and drinks, solar energy, nuclear energy, energy from petrol, and so on. But how is energy defined by physicists? It is observed in a large number of situations that the energy of an object allows it to do work. So, generally energy is defined as follows.

The capacity of an object to do work is called the energy of the object

The capacity to do work can come through a number of ways. One important way to acquire this capacity is through motion. When a body is moving, it has more energy than when it is at rest.

The values of $\cos\theta$ for some angles are being given here

$\cos 0° = 1$ $\quad\cos 30° = \frac{\sqrt{3}}{2}$ $\quad\cos 45° = \frac{1}{\sqrt{2}}$ $\quad\cos 60° = \frac{1}{2}$ $\quad\cos 90° = 0$

Another way for an object to acquire energy is to be at special positions or in special shapes. There are other ways of acquiring energy too.

Kinetic Energy

Consider a ball kept in contact with a block on a table (Figure 5.6a). The ball and the block stay at rest and no work is done by the ball on the block. Now suppose, the ball is separated from the block and is moved towards it with a speed v (Figure 5.6b) Also, suppose that the ball sticks to the block as it hits it. The ball pushes the block forward and the block is displaced. So, the moving ball can do work on the block whereas the same ball, when at rest, cannot do any work.

(a) (b)

F5-06

Fig. 5.6

We conclude that a moving ball has more capacity to do work than an otherwise similar ball at rest. In other words, a body has more energy when it is moving than when it is at rest.

The energy of an object because of its motion is called its kinetic energy.

A flying bird, a running man, a moving train and a swinging bat are some examples of bodies with kinetic energy.

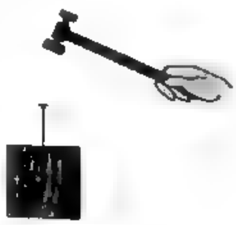


Fig 5.7

Let us consider some more examples of the ability of objects with kinetic energy to do work. Suppose you want to drive a nail into a wooden block. You bring down the hammer on the nail at a high speed. Because of its motion, the hammer has kinetic energy. This energy allows the hammer to do work on the nail, i.e., to move it through a distance.

When a small stone falls on soft soil or mud, it makes a small depression in the soil. The kinetic energy of the stone allows it to do work, which in this case is to displace the soil. A large falling stone or a stone falling with high speed has greater kinetic energy than a small stone falling from a height of a few metres. Such a stone will displace more soil.

Sometimes a large rock from space hits the earth's surface at a very high speed. Its huge kinetic energy creates a large crater on the earth's surface. One such crater was formed thousands of years ago at Lonar in Maharashtra. The diameter of this crater is 1.8 km.

Expression for Kinetic Energy

Fig. 5.8

How do we define a numerical value of kinetic energy? We take the kinetic energy of a body at rest to be zero. To bring it in motion, i.e., to give it kinetic energy, some work has to be done on it. It is reasonable to assume that the amount of work done on an object is the same as the increase in its kinetic energy if there is no other change in the object. Suppose a body of mass m is kept at rest on a smooth horizontal surface at A (Figure 5.8). A force F starts acting on it in the horizontal direction and keeps acting on it till the body reaches B. The distance $AB = x$.

The acceleration of the body is

$$a = \frac{F}{m}$$

The velocity v at B may be determined from the equation

$$v^2 = u^2 + 2as.$$

Here, the initial velocity (at A) is $u = 0$. So,

$$v^2 = 2ax$$

or $$v^2 = 2\frac{F}{m}x$$

or $$Fx = \frac{1}{2}mv^2. \qquad \text{...(i)}$$

But Fx is the work done by the force on the body It should be equal to the increase in the kinetic energy of the body as it moves from A to B. Also, since the kinetic energy at A was zero, the increase in kinetic energy should be equal to the kinetic energy at B. So, we conclude the following.

The kinetic energy of a body of mass m moving with a speed v is $\frac{1}{2}mv^2$.

$$\textit{Kinetic energy} = \frac{1}{2}mv^2$$

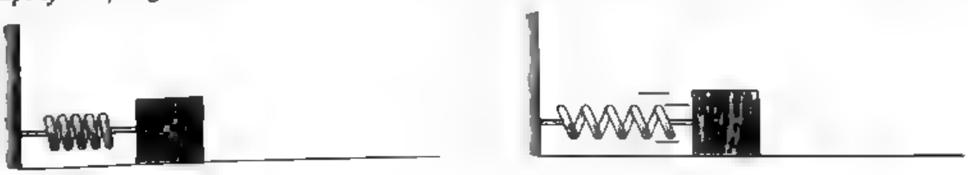
EXAMPLE 5.[illegible] **Find the kinetic energy of a ball of mass 200 g moving at a speed of 20 cm/s.**

Solution The kinetic energy is

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(0.200 \text{ kg}) \times (0.20 \text{ m/s})^2 = 0.004 \text{ J}.$$

Potential Energy

An object can also acquire the capacity to do work because of its position or shape. As an example, suppose a spring is fixed to a wall, and a block is compressed against the spring and then released (Figure 5.9a). The spring pushes the block away (Figure 5.9b), and it does work on the block in the process. If we place the block in contact with the spring at its natural length, the spring does not push it, and hence, does not do any work. We conclude that a compressed spring has more energy than a spring at its natural length. *This extra energy is due to the special (compressed) shape of the spring.*



(a) (b)

Fig 5.9

Some toy cars have a coiled spring inside them. This spring is compressed (tightened) either by winding a key or by pulling the car backwards. The energy of the spring increases when it is compressed. When the car is released, this energy drives the car forward.

As another example, consider the case of a stone falling on soft soil and creating a depression in it. First the stone is raised to some height above the ground. Then it is allowed to fall. As it falls, it gains speed, which allows it to do work on the soil. But while lying on the ground, the stone cannot do any work. So, the stone when placed at a height above the ground has a greater capacity to do work (i.e., it has greater energy) than when it is on the ground. *This greater energy is due to the special position (height) of the stone.*

The energy of an object because of its position or shape is called its potential energy.

Gravitational potential energy

We have seen that when an object is raised to a height, it acquires potential energy. The potential energy due to height above the earth's surface is called gravitational potential energy.

Consider two positions of a ball of mass *m*, one at *A* and the other at *B* (Figure 5.10). The position *B* is at a height $h = AB$ above the position *A*.

Suppose someone brings down the ball from *B* slowly. The ball exerts a force mg on the hand in the vertically downward direction. As the hand is displaced from *B* to *A*, the force exerted by the ball on the hand does some work. This work is

$$W = Fd = mgh.$$

Thus, the ball does work mgh in coming to *A*. This means that the capacity of the ball to do work was greater at *B* than that at *A* by mgh. In other words, the ball at *B* has a gravitational potential energy mgh more than that at *A*.

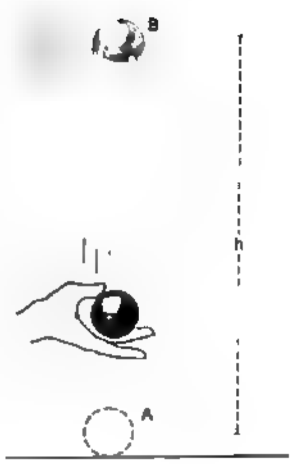


Fig. 5.10

If the gravitational potential energy at *A* is U_A and that at *B* is U_B,

$$U_B - U_A = mgh$$

or $$U_B = U_A + mgh.$$

This gives us the difference in gravitational potential energies between the two positions. But what is the potential energy at *A*? You can choose any position, and say that the potential energy is zero there. You can choose the potential energy at *A* to be zero. Then the potential energy at *B* is $U_B = mgh$. If you take the potential energy at *B* to be zero, the potential energy at *A* is $U_A = -mgh$.

In general, if the potential energy at the ground is taken as zero, the potential energy of an object at a height *h* above the gound is given by

$$\boxed{U = mgh} \quad \text{... 5.4}$$

This energy results from the force of attraction mg between the earth and the object. From Newton's third law, both the earth and the object attract each other. Hence, strictly speaking, the energy mgh is not the potential energy of the object alone—it is the potential energy of the object–earth system.

EXAMPLE 5.5 **An object of mass 1 kg is raised through a height *h*. Its potential energy is increased by 1 J. Find the height *h*.**

Solution The potential energy is increased by mgh. Thus,

$$mgh = 1\text{ J}$$

or $$(1\text{ kg}) \times (9.8\text{ m/s}^2)\,h = 1\text{ J}$$

or $$h = \frac{1\text{ J}}{(1\text{ kg}) \times (9.8\text{ m/s}^2)} = 0.102\text{ m}$$

Equation 5.4 was derived by assuming that the weight of the body remains constant at the value mg during the displacement. This is quite acceptable if we are talking of displacements which are small in comparison to the radius of the earth (6,400 km). Thus, there is no difficulty in using Equation 5.4 in situations such as a ball thrown in a field, a stone falling from the top of a building, etc. However, for objects which move through large vertical distances (say, thousands of kilometres) above the earth's surface, we cannot use this equation to calculate potential energy.

Elastic potential energy

When a spring is stretched or compressed from its natural length, it gets extra energy. It can return to its natural length by performing some work. The extra energy stored in a stretched or compressed spring is called its elastic potential energy. A stretched rubber band also has elastic potential energy, whereas a rubber band at its natural length lying on a table has no elastic potential energy.

Do you know how a catapult (*gulel*) works? It has a thick rubber band whose two ends are fixed to the arms of the catapult. The band is pulled while holding a small object in its fold. The length of the band increases. When it is released, it regains its natural length. While doing so, it accelerates the object, which leaves the catapult at a great speed.

When the band is stretched, it acquires elastic potential energy. When it is released, it does work on the object to accelerate it.

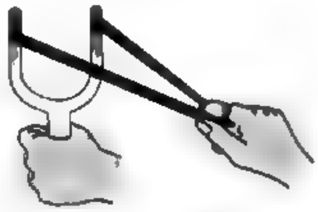


Fig. 5.11

Mechanical Energy

The sum of the kinetic energy and the potential energy of an object is called its mechanical energy

The kinetic energy of an aeroplane at rest is zero. We can choose its potential energy to be zero on the ground. Then, its mechanical energy is zero while it rests on the ground. When the same aeroplane flies, it has kinetic as well as potential energy. Their sum gives the aeroplane's mechanical energy in flight.

Work and Energy

Whenever work is done on an object, its energy increases. When we push a block kept on a table, the block starts moving. We do work on the block and the block acquires kinetic energy.

Some people use a device called bullworker to build muscles. They compress it by exerting inward forces at its ends. Each of these forces does work on the bullworker, making the ends move inwards. As a result, the spring in the bullworker is compressed and its elastic potential energy is increased. We can say that the work done on this system is stored in it as energy.

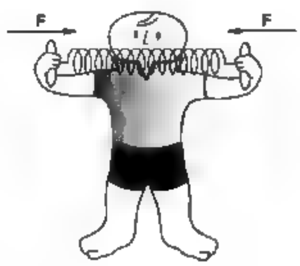


Fig 5.12

Work done by external forces on a system is equal to the increase in the system's energy.

If negative work is done on a system, its energy decreases. When a ball moves on the ground, the force of friction does negative work on the ball. This is because the force of friction acts backwards, while the displacement of the ball is in the forward direction. Because of the negative work, the kinetic energy of the ball decreases and it comes to a stop.

Other Forms of Energy

Besides mechanical energy, energy can exist in several other forms. Heat, light, sound, etc., are forms of energy. Charged particles and electric currents can produce electrical energy and magnetic energy. Electric batteries, cooking gas, petrol, etc., have chemical energy stored in them. Even matter itself is a concentrated form of energy and can be converted into other forms of energy such as kinetic energy and heat energy.

Energy inside the Human Body

The human body is a complicated system in which a variety of energy changes take place. A number of activities take place inside the body—some increase the energy of the body and some reduce it. The main source of energy for the body is food. How weak you feel if you do not eat for

a day! Energy is used up when the muscles are stretched, the heart pumps blood, and so on. Work is done by the forces inside the body in such activities, and hence, the energy is reduced. We feel tired when the energy of the body is used up beyond a limit.

Fig. 5.13

Do you know why a person gets tired while standing at a place for a long time. Seen from the outside, there might not be any visible motion. So, no work is done by the person on any external object. But there is motion inside the body. When you stand for a long time, the muscles of your body are stretched and the heart has to pump more blood to them. Several chemical reactions take place inside the body to sustain these activities. Work is done in all the internal motions and there is a corresponding loss of energy. If the person has to keep a heavy load on his/her hand, the internal work done is more, and he/she gets tired more quickly.

CONSERVATION OF ENERGY

Energy can neither be created nor destroyed. It can only be changed from one form to another.

The above principle is called the principle of conservation of energy It means that when all forms of energy are taken into account, the total energy in the universe remains constant. The principle is also valid for an isolated system, i.e., one on which no force acts from outside.

When a ball is dropped from a height, its gravitational potential energy decreases, since its height decreases. It may seem that energy has been lost. But the ball gains speed, and hence, it acquires kinetic energy. Actually, as the ball falls its potential energy is gradually converted into kinetic energy

When we switch on an electric torch, the chemical energy stored in the batteries is converted into heat energy as the filament of the bulb gets heated. Heat energy is then converted into light energy.

When a person does work on an object, the energy of the object increases. As the total energy is constant, the person must lose the same amount of energy. That is why when we push a heavy box on a surface, we feel tired. We do work on the box, and hence, lose energy.

Have you seen a simple pendulum? You can easily make one for yourself. Take a long thread, say, 50–60 cm long and attach a small, heavy object at one end. Tie the other end to a nail fixed in the wall. Your simple pendulum is ready. Pull the object (called the bob of the pendulum) to one side, to the position *B* (Figure 5.14), and release it. What do you see? The bob swings towards the opposite side, reaches the point C and returns. It repeats this motion over and over again. At the extreme positions *B* and *C*, the height of the bob is maximum, and at the midpoint (*A*) it is the minimum. So, the potential energy is the greatest at *B* or *C*, and is the least at *A*. On the other hand, the bob stops momentarily at the extreme positions *B* and *C*. And it moves with the greatest speed when it crosses the midpoint *A*. So, as the bob moves from *B* to *A*, its potential energy decreases and its kinetic energy increases. And as it moves from *A* to *C*, its potential energy increases and its kinetic energy decreases. Neglecting a small loss of energy due to air resistance, the sum of the potential energy and kinetic energy remains constant.

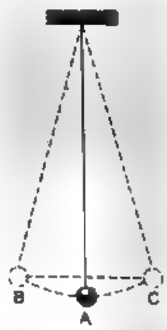


Fig. 5.14

Conservation of Energy in a Freely Falling Body

Suppose a ball of mass m is kept at rest at a point A, which is at a height h above the ground (Figure 5 15) We take the potential energy at the ground to be zero. The potential energy of the ball at A is then

$$U_A = mgh.$$

The ball is at rest at A, so its kinetic energy here is

$$K_A = 0.$$

The total energy is

$$U_A + K_A = mgh. \qquad \ldots(i)$$

The ball is now allowed to fall. The only force on it during the fall is gravity (free fall), and hence, its acceleration is $a = g$ downwards. Consider any point B in its path. Suppose its distance from A is x. The height of the point B from the ground is $(h - x)$. When the ball reaches point B, its potential energy is

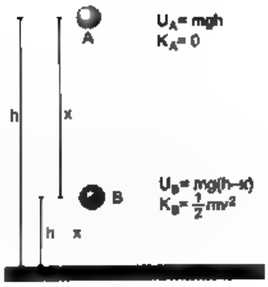


Fig. 5.15

$$U_B = mg(h - x).$$

The velocity of the ball at point B is given by

$$v^2 = u^2 + 2gx = 2gx.$$

The kinetic energy of the ball at point B is

$$K_B = \frac{1}{2}mv^2 = \frac{1}{2}m(2gx) - mgx.$$

The total energy at point B is

$$U_B + K_B = mg(h - x) + mgx = mgh. \qquad \ldots(ii)$$

Comparing (i) and (ii), we see that

$$U_A + K_A = U_B + K_B.$$

The point B was chosen to be any point in the path of the ball. The total energy here turned out to be equal to the initial energy. Thus, the energy at all points during the fall is the same, that is, remains conserved as the ball falls. The potential energy is gradually converted into kinetic energy.

This result is also true if a ball is thrown upwards or at some angle with the vertical. During the upward movement of a ball, its kinetic energy gets gradually converted to potential energy.

Examples of Conversion of Energy

Life on the earth depends on the energy received from the sun Hydrogen nuclei (protons) fuse together to form helium nuclei in the sun's core. In this process, energy of the nuclei is converted into heat energy. This heat energy is absorbed by the atoms at the surface of the sun, and a part of it is converted into light and other radiations. These radiations travel through millions of kilometres of empty space to reach the earth. On receiving radiation from the sun, the land and air get heated. This, as you know, causes wind. This means that heat energy gets converted into kinetic energy. The energy from the sun also heats up the waters of the oceans. Water evaporates from the oceans and rises up to form clouds. This is a conversion of kinetic energy into potential energy.

We see many other energy conversions in nature. Snow deposited at high altitudes melts and the water so formed flows down to the seas. In the process, the potential energy of the water is converted into kinetic energy. We convert this kinetic energy of water to electrical energy in hydroelectric power plants.

Plants use sunlight for photosynthesis. In this process, light energy gets converted to chemical energy (as the energy stored in plant food). When plants die, the energy stored in them is not lost. For example, dead plants buried below the earth's surface for millions of years got converted to fuels such as coal, petroleum and gases. These have chemical energy stored in them. When these fuels are burnt, the chemical energy is converted into heat energy.

The food we eat comes from plants and from animals that eat plants or plant products. When we eat food, several chemical processes take place, and energy in the form needed by the body is produced from the energy contained in the food. When we walk, run, exercise, etc., this energy is used to provide kinetic energy.

POWER

You may have noticed that old people find it difficult to climb a flight of stairs quickly. But they can easily climb the same flight of stairs slowly. In both cases they spend the same amount of their stored energy and do the same amount of work in climbing the stairs. But the rate at which energy is spent or work is done is different in the two cases.

Let us take another example. You can keep your hands immersed in lukewarm water for hours. But you cannot keep them in boiling water even for a second. The total transfer of energy from water to your hands may be the same in both the cases. However, the rate at which energy is transferred is different in the two cases. As a result, they have very different effects.

We see that in many cases the rate at which work is done or energy is transferred is an important quantity. Hence, this quantity is given a separate name—power.

The rate at which work is done by a force is called the power delivered by that force.

Also, the rate at which energy is transferred by an object (like a machine) is called the power delivered by that object.

If a force does work W in time t, the average power delivered by the force is

$$P = \frac{W}{t} \qquad \text{...5.5}$$

If the force does work at a constant rate, the average power is the same as the power at any instant during the time the work is being done.

As work W is measured in joules and time in seconds, the unit of power is joule/second. This unit is given a separate name—watt. If work is done at the rate of 1 joule per second, the power delivered is 1 watt **(1 watt = 1 joule/second).** The symbol of watt is W. Kilowatt (kW) and megawatt (MW) are also commonly used units of power.

$$1\ \text{kW} = 1000\ \text{W}$$

$$1\ \text{MW} = 1000{,}000\ \text{W} = 10^6\ \text{W}.$$

Another common unit of power is the horsepower. Horsepower is related to watt as

$$\textbf{1 horsepower = 746 W} \qquad \text{...5.6}$$

Horsepower is written in short as hp.

EXAMPLE 5.6 **A man carrying a bag of mass 25 kg climbs up to a height of 10 m in 50 seconds. Calculate the power delivered by him to the bag.**

Solution The force exerted by the man on the bag is equal to the weight of the bag, which is

$$mg = (25\ \text{kg}) \times (9.8\ \text{m/s}^2)$$
$$= 245\ \text{N}.$$

The work done by this force in taking the bag up by 10 m is

$$W = (245\ \text{N}) \times (10\ \text{m})$$
$$= 2{,}450\ \text{J}.$$

This work is done in 50 s. The power delivered is

$$P = \frac{W}{t}$$
$$= \frac{2450\ \text{J}}{50\ \text{s}} = 49\ \text{W}.$$

We use electric bulbs rated at 100 W, 60 W, etc. The rated value gives the electric energy (U) consumed by the bulb per unit time and is called its power ($P = U/t$). A 100-W bulb consumes 100 J of electric energy per second. A 1-mW (milliwatt) laser beam delivers 1 mJ of energy every

second in the form of light to the surface on which it falls. We say that the power of the laser beam is 1 mW A 100-MW (megawatt) power plant produces 100 MJ of electric energy per second.

Commercial Unit of Energy

The SI unit of energy is the joule (J). But the commonly used unit for electrical-energy consumption is the kilowatt-hour (kWh) If energy is consumed at the rate of 1 kilojoule/second, i.e., at the rate of 1 kilowatt, and this continues for 1 hour then the total energy consumed is called 1 kilowatt-hour.

Thus,
$$\begin{aligned} 1\ \text{kWh} &= 1\text{kW} \times 1\ \text{hour} \\ &= (1000\ \text{W}) \times (3600\ \text{s}) \\ &= (1000\ \text{J/s}) \times (3600\ \text{s}) \\ &= (3600000\ \text{joules}) = 3.6 \times 10^6\ \text{J}. \end{aligned}$$

$$\boxed{1\ \text{kWh} = 3.6 \times 10^6\ \text{J}}$$

For electrical-energy consumption in houses, factories, shops, etc., kilowatt-hour is simply called 'unit'. So, the cost of electrical energy is given in terms of Rs/unit. If electricity costs Rs 5.00/unit, it means that consumers pay Rs 5.00 for every unit (kWh) of electrical energy consumed.

SOLVED PROBLEMS

EXAMPLE 1 **Calculate the work done by a student in lifting a 0.5-kg book from the ground and keeping it on a shelf 1.5 m high.**

Solution The weight of the book is

$$mg = (0.5\ \text{kg}) \times (9.8\ \text{m/s}^2) = 4.9\ \text{N}.$$

The student has to apply an equal force in the upward direction to move it up. The displacement is 1.5 m in the direction of the force. Thus, the work done is

$$\begin{aligned} W &= Fd \\ &= (4.9\ \text{N}) \times (1.5\ \text{m}) = 7.35\ \text{J}. \end{aligned}$$

EXAMPLE 2 **A roller is pushed with a force of 100 N along its handle, which is at an angle of 30° with the horizontal. Find the work done in moving it through 10 m.**

Solution The work done is

$$\begin{aligned} W &= Fd\cos\theta \\ &= (100\ \text{N}) \times (10\ \text{m}) \cos 30^\circ = (100\ \text{N}) \times (10\ \text{m}) \times \frac{\sqrt{3}}{2} = 500\sqrt{3}\ \text{J}. \end{aligned}$$

EXAMPLE 3 **A block of mass 1 kg slides down on an inclined plane of inclination 30°. Find the work done by the block's weight as it slides through 50 cm.**

Solution Suppose the block slides from A to B (Figure 5.W1). The displacement is AB = 50 cm = 0.5 m along the inclined plane. The weight is

$$(1\ \text{kg}) \times (9.8\ \text{m/s}^2) = 9.8\ \text{N in the downward direction.}$$

The angle between the force and the displacement is 60° as shown in the figure. Hence, the work done by the weight is

$$\begin{aligned} &(9.8\ \text{N}) \times (0.50\ \text{m}) \times (\cos 60^\circ) \\ &= (9.8\ \text{N}) \times (0.50\ \text{m}) \times \frac{1}{2} = 2.45\ \text{J}. \end{aligned}$$

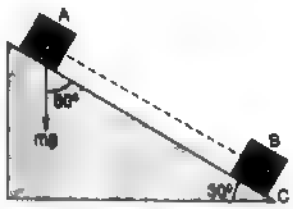


Fig. 5.W1

EXAMPLE 4 **A force of 10 N displaces an object through 20 cm and does work of 1 J in the process. Find the angle between the force and the displacement.**

Solution We have

$$W = Fd\cos\theta$$

$$= (10\ \text{N}) \times (20\ \text{cm})\cos\theta$$

or $$1\ \text{J} = (10\ \text{N}) \times (0.2\ \text{m})\cos\theta$$

$$= (2\ \text{J})\cos\theta$$

or $$\cos\theta = \frac{1}{2}.$$

$\because\ \cos 60° = \frac{1}{2}\qquad \therefore\ \theta = 60°.$

EXAMPLE 5 **A player kicks a ball of mass 250 g placed at the centre of a field. The ball leaves his foot with a speed of 8 m/s. Find the work done by the player on the ball.**

Solution Initially, the ball is at rest. The player does some work on it, and hence, the ball gains kinetic energy. So, the work done by the player on the ball is equal to the kinetic energy of the ball as it leaves the foot.

$$W = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(0.25\ \text{kg})(8\ \text{m/s})^2 = 8\ \text{J}.$$

EXAMPLE 6 **A 1.0-kg ball is thrown up with a speed of 9.9 m/s. Calculate the work done by its weight in one second.**

Solution The distance moved in one second is

$$s = ut - \frac{1}{2}gt^2$$

$$= (9.9\ \text{m/s}) \times (1\ \text{s}) - \frac{1}{2} \times (9.8\ \text{m/s}^2) \times (1\ \text{s})^2$$

$$= 5\ \text{m}.$$

The weight of the ball is $mg = (1.0\ \text{kg}) \times (9.8\ \text{m/s}^2) = 9.8\ \text{N}$ in the downward direction.
The work done in one second is

$$-(9.8\ \text{N}) \times (5\ \text{m}) = -49\ \text{J}.$$

EXAMPLE 7 **A 10-kg ball is thrown upwards with a speed of 5 m/s. (a) Find its potential energy when it reaches the highest point. (b) Calculate the maximum height it reaches.**

Solution (a) The kinetic energy of the ball is

$$K = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times (10\ \text{kg}) \times (5\ \text{m/s})^2 = 125\ \text{J}.$$

At the highest point, the kinetic energy becomes zero, and hence, the entire kinetic energy of 125 J is converted into potential energy. So, the potential energy at the highest point is 125 J.

(b) Suppose the ball reaches a maximum height h. Its potential energy there will be mgh. Thus,

$$mgh = 125\ \text{J}$$

or $$h = \frac{125\ \text{J}}{(10\ \text{kg}) \times (9.8\ \text{m/s}^2)}$$

$$= 1.28\ \text{m}.$$

EXAMPLE 8 **A 10-kg ball is dropped from a height of 10 m. Find (a) the initial potential energy of the ball, (b) the kinetic energy just before it reaches the ground, and (c) the speed just before it reaches the ground.**

Solution

(a) At the height of 10 m, the potential energy is

$$mgh = (10\ \text{kg}) \times (9.8\ \text{m/s}^2) \times (10\ \text{m}) = 980\ \text{J}.$$

(b) As it reaches the ground, the potential energy becomes zero. So, just before reaching the ground the entire potential energy is converted into kinetic energy. The kinetic energy just before the ball reaches the ground is, therefore, 980 J.

(c) Let the speed of the ball just before it reaches the ground be v.
The kinetic energy is

$$K = \frac{1}{2}mv^2$$

or

$$980\ \text{J} = \frac{1}{2} \times (10\ \text{kg})\, v^2$$

or

$$v^2 = 196\ \text{J/kg} = 196\ \text{m}^2/\text{s}^2$$

or

$$v = 14\ \text{m/s}.$$

[illegible] **A body A of mass 3.0 kg and a body B of mass 10 kg are dropped simultaneously from a height of 14.9 m. Calculate (a) their momenta, (b) their potential energies, and (c) their kinetic energies when they are 10 m above the ground.**

Solution

(a) As the two bodies are dropped, they fall with the same acceleration of 9.8 m/s^2. When they are 10 m above the ground, they have already fallen through 14.9 m – 10 m = 4.9 m. The velocity at this point may be worked out from

$$v^2 = u^2 + 2gh$$

$$= 0 + 2 \times (9.8\ \text{m/s}^2) \times (4.9\ \text{m})$$

$$= 9.8 \times 9.8\ \text{m}^2/\text{s}^2$$

or

$$v = 9.8\ \text{m/s}.$$

The momentum of A is

$$p_A = m_A v$$

$$= (10\ \text{kg}) \times (9.8\ \text{m/s}) = 29.4\ \text{kg m/s},$$

and that of B is

$$p_B = m_B v$$

$$= (10\ \text{kg}) \times (9.8\ \text{m/s}) = 98\ \text{kg m/s}.$$

(b) The bodies are at a height of 10 m above the ground. The potential energy of A is

$$U_A = m_A gh$$

$$= (3.0\ \text{kg}) \times (9.8\ \text{m/s}^2) \times (10\ \text{m}) = 294\ \text{J}.$$

and that of B is

$$U_B = m_B gh$$

$$= (10\ \text{kg}) \times (9.8\ \text{m/s}^2) \times (10\ \text{m}) = 980\ \text{J}.$$

(c) The kinetic energy of A is

$$K_A = \frac{1}{2} m_A v^2$$

$$= \frac{1}{2} \times (3.0\ \text{kg}) \times (9.8\ \text{m/s})^2 \approx 144\ \text{J},$$

and that of B is

$$K_B = \frac{1}{2} m_B v^2$$

$$= \frac{1}{2} \times (10\ \text{kg}) \times (9.8\ \text{m/s})^2 \approx 480\ \text{J}.$$

[illegible] **A block of mass 2.0 kg slides on a rough surface. At $t = 0$, its speed is 2.0 m/s. It stops after covering a distance of 20 cm because of the friction exerted by the surface on it. Find the work done by friction.**

Solution The kinetic energy of the ball at $t = 0$ is

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times (2.0 \text{ kg}) \times (2.0 \text{ m/s})^2 = 4 \text{ J}.$$

When the block comes to rest, its kinetic energy becomes zero. This loss of energy takes place because friction does negative work on the block. Thus, the work done by the friction is (– 4 J).

EXAMPLE 11 **A ball is dropped from a height H. When it reaches the ground, its velocity is 40 m/s. Find the height H.**

Solution The potential energy of the body initially at rest at height H is entirely converted into kinetic energy when it reaches the ground.

$$\frac{1}{2}mv^2 = mgH$$

or

$$H = \frac{v^2}{2g}$$

$$= \frac{40 \times 40 \text{ m}^2/\text{s}^2}{2 \times 9.8 \text{ m/s}^2} = 81.6 \text{ m}.$$

EXAMPLE 12 **How much time will it take to perform 440 J of work at a rate of 11 W?**

Solution We have

$$P = \frac{W}{t}$$

or

$$t = \frac{W}{P} = \frac{440 \text{ J}}{11 \text{ W}} = 40 \text{ s}.$$

EXAMPLE 13 **A man does 200 J of work in 10 seconds and a boy does 100 J of work in 4 seconds. (a) Who is delivering more power? (b) Find the ratio of the power delivered by the man to that by the boy.**

Solution (a) Power delivered by the man is

$$P_{\text{man}} = \frac{200 \text{ J}}{10 \text{ s}} = 20 \text{ W},$$

and that by the boy is

$$P_{\text{boy}} = \frac{100 \text{ J}}{4 \text{ s}} = 25 \text{ W}.$$

So, the boy is delivering more power.

(b) The ratio is

$$\frac{P_{\text{man}}}{P_{\text{boy}}} = \frac{20}{25} = \frac{4}{5}.$$

EXAMPLE 14 **Five 100-W bulbs are used for 10 hours every day for 30 days. Find the cost of electricity if the rate is Rs 4.00/unit.**

Solution For each bulb, power $= P = 100 \text{ W} = \frac{100}{1000}$ kW.

The energy consumed by a bulb every day $= U = Pt = \frac{100}{1000} \text{ kW} \times 10 \text{ h} = 1 \text{ kWh}$

$\therefore$ the energy consumed by 5 bulbs in 30 days = 5 × 1 kWh × 30 = 150 kWh.

The cost of electricity at the rate of Rs 4.00/kWh is

$$(\text{Rs } 4.00/\text{kWh}) \times (150 \text{ kWh}) = \text{Rs } 600.00.$$

• POINTS TO REMEMBER •

- *Work*
  - (a) When a force acts on an object and the object moves, work is done by the force on the object.
  - (b) The work done W by a force F on an object is
    $W = Fd \cos \theta$,
    where d is the displacement of the object, and θ is the angle between the force and the displacement.
  - (c) If the displacement is along the force, $W = Fd$.
    If the displacement is opposite to the force, $W = -Fd$.
    If the displacement is perpendicular to the force, the work done is zero.
- *Energy*
  The capacity of an object to do work is called its energy.
  The energy of an object because of its motion is called its kinetic energy.
  The energy of an object because of its position or shape is called its potential energy.
  The sum of the kinetic energy and the potential energy of an object is called its mechanical energy.
- *Gravitational potential energy*
  The potential energy of an object due to its height above the earth's surface is called its gravitational potential energy.
- *Elastic potential energy*
  A stretched or compressed spring has elastic potential energy. A stretched rubber band also has elastic potential energy.
- *Relation between work and energy*
  Work done by external forces on a system is equal to the increase in its energy.
- *Principle of conservation of energy*
  Energy can neither be produced nor destroyed. It can only be changed from one form to another.
- Functions inside the human body such as the stretching of muscles, pumping of blood by the heart, etc., involve work and loss of energy. When our body expends energy beyond a limit, we feel tired
- *Power*
  The rate at which work is done by a force is called the power delivered by that force.
- 1 horsepower = 746 W
  1 kWh = 3.6×10^6 J
- *Mathematical equations*

Work	$W = Fd \cos \theta$
Gravitational potential energy	$U = mgh$
Kinetic energy	$K = \frac{1}{2} mv^2$
Power	$P = W/t$

• EXERCISES •

A. *Objective Questions*

I. *Pick the correct option.*

1. The work done by the weight of a 1-kg mass while it moves up through 1 m is
   (a) 9.8 J (b) −9.8 J (c) $\frac{1}{9.8}$ J (d) $-\frac{1}{9.8}$ J
2. A force of 10 N acts on a body towards the east. The work done by the force while the body moves through 1 m in the east-north direction (midway between east and north) is
   (a) $10\sqrt{2}$ J (b) $\frac{10}{\sqrt{2}}$ J (c) −10 J (d) $-\frac{10}{\sqrt{2}}$ J
3. When a stone tied to a string is whirled in a circle, the work done on it by the string is
   (a) positive (b) negative (c) zero (d) undefined
4. A man with a box on his head is climbing up a ladder. The work done by the man on the box is
   (a) positive (b) negative (c) zero (d) undefined
5. A porter with a suitcase on his head is climbing up a flight of stairs with a uniform speed. The work done by the 'weight of the suitcase' on the suitcase is
   (a) positive (b) negative (c) zero (d) undefined
6. As a body rolls down an inclined plane, it has
   (a) only kinetic energy
   (b) only potential energy
   (c) both kinetic energy and potential energy
   (d) neither kinetic energy nor potential energy
7. The kinetic energy of a body depends
   (a) on its mass only
   (b) on its speed only
   (c) on its mass as well as on its speed
   (d) neither on its mass nor on its speed
8. In which of the following cases is the potential energy of a spring minimum?
   (a) When it is compressed
   (b) When it is extended

(c) When it is at its natural length
(d) When it is at its natural length but is kept at a height h above the ground

9. A particle of mass 100 g moves at a speed of 1 m/s. Its kinetic energy is
(a) 50 J (b) 5 J (c) 0.5 J (d) 0.05 J

10. A ball is thrown upwards from a point A. It reaches up to the highest point B and returns.
(a) Kinetic energy at A = kinetic energy at B
(b) Potential energy at A = potential energy at B
(c) Potential energy at B = kinetic energy at B
(d) Potential energy at B = kinetic energy at A

11. A body of mass 2 kg is dropped from a height of 1 m. Its kinetic energy as it reaches the ground is
(a) 19.6 J (b) 19.6 N
(c) 19.6 kg (d) 19.6 m

12. Two bodies of unequal masses are dropped from a cliff. At any instant, they have equal
(a) momentum (b) acceleration
(c) potential energy (d) kinetic energy

13. When the speed of a particle is doubled, its kinetic energy
(a) remains the same (b) gets doubled
(c) becomes half (d) becomes 4 times

14. When the speed of a particle is doubled, the ratio of its kinetic energy to its momentum
(a) remains the same (b) gets doubled
(c) becomes half (d) becomes 4 times

15. The unit of power is
(a) watt (b) joule (c) newton (d) kg

16. A person A does 500 J of work in 10 minutes and another person B does 600 J of work in 20 minutes. Let the power delivered by A and B be P_1 and P_2 respectively. Then,
(a) $P_1 = P_2$
(b) $P_1 > P_2$
(c) $P_1 < P_2$
(d) P_1 and P_2 are undefined

II. *Mark the statements true (T) or false (F).*

1. Work and energy have different units.
2. A stone tied to a string is whirled in a circle. The work done by the weight of the stone for a small displacement is zero.
3. When an aeroplane takes off, the work done by its weight is positive.
4. The potential energy of a spring increases when it is extended and decreases when it is compressed.
5. When negative work is done by external forces on a system, the energy of the system decreases.
6. A person stands for a long time. Work is done by forces operating inside his body.
7. When a body falls, its kinetic energy remains constant.
8. A rubber band has more potential energy when wrapped around a packet than when it was lying unused.

III. *Fill in the blanks.*

1. When force and displacement are in perpendicular directions, the work is ....
2. The unit of work in terms of newton and metre is ....
3. A force of 20 N acts on an object, which is displaced by 2 m. The direction of the displacement is not known. The maximum and minimum amounts of work possible are respectively ...... and ....
4. The sum of kinetic energy and potential energy is called ....
5. When an object is thrown up, its potential energy ...... and its kinetic energy ...... till it reaches the highest point.
6. When an object is dropped from a height, its potential energy ...... and its kinetic energy ...... till it reaches the ground.
7. A compressed spring has ...... potential energy than the potential energy which it has at the natural length
8. When a torch is switched on, the ...... energy of the batteries is converted into heat and light.

B. *Very-Short-Answer Questions*

Answer the following in one word or maximum one sentence.

1. Does work have a direction?
2. State the expression for work done by a force when the object experiencing the force is displaced opposite to the force.
3. Does the kinetic energy of an object depend on its direction of motion?
4. The displacement of a particle makes an angle θ with the force acting on it. For which value of θ, is the work done maximum? For which value of θ is the work done minimum?
5. By what factor does the kinetic energy of a particle increase if the speed is increased by a factor of 3?
6. State the expression for the kinetic energy of a particle of mass m moving at a speed v.
7. What is mechanical energy?
8. What is the relation between the work done by external forces on a body and its energy?
9. Can matter be converted into energy?
10. The work done by a force F_1 is larger than the work done by another force F_2. Is it necessary that power delivered by F_1 is also larger than the power delivered by F_2?

C. *Short-Answer Questions*

Answer the following in about 30–40 words each.

1. In which of the following cases is the work done positive, zero or negative?
(a) Work done by a porter on a suitcase in lifting it from the platform on to his head

(b) Work done by the force of gravity on a suitcase as the suitcase falls down from the porter's head

(c) Work done by a person on a book held in his hand while walking with uniform speed on a horizontal road

2. Define work and write its unit.
3. Give an example where the displacement of a particle is in the direction opposite to a force acting on this particle.
4. Give an example where the displacement of a particle is in the direction perpendicular to a force acting on this particle.
5. Define energy and state its unit.
6. What do you understand by potential energy? Explain with two examples.
7. Give two examples of elastic potential energy.
8. State the principle of conservation of energy.
9. Give an example of conversion of chemical energy into heat energy.
10. Write the names of four different forms of energy and give an example where energy is transformed from one form into another.
11. Define power and write its unit.

D. *Long-Answer Questions*

Answer the following in not more than 70 words.

1. Show that the work done by a force is given by the product of the force and the component of the displacement along the force.
2. Give an example to show that a body can have larger energy because of its special shape.
3. Find the expression for the gravitational potential energy of a body of mass m at a height h.
4. How can you justify that a body kept at a greater height has larger energy?
5. Find the expression for kinetic energy of a body of mass m moving at a speed v.
6. Why does a person standing for a long time get tired when he does not appear to do any work?
7. What do you understand by potential energy? Explain with two examples.

E. *Numerical Problems*

1. Calculate the work done by a person in lifting a load of 20 kg from the ground and placing it on a 1-m-high table.
2. A 2-kg body is sliding up an inclined plane of inclination 30°. Find the work done by the force of gravity in moving the body through 1 m.
3. A force of 12 N does a work of 12 J on a body when the body is displaced through 2 m. At what angle to the displacement does the force act?
4. Find the mass of a body which has 5 J of kinetic energy while moving at a speed of 2 m/s.
5. Calculate the speed of a body of mass 200 g having a kinetic energy of 10 J.
6. A body of mass 25 g has a momentum of 0.40 kg m/s. Find its kinetic energy.
7. The mass of a ball A is double the mass of another ball B. The ball A moves at half the speed of the ball B. Calculate the ratio of the kinetic energy of A to the kinetic energy of B.
8. Calculate the increase in potential energy as a block of 2 kg is lifted up through 2 m.
9. A ball of mass 0.5 kg is dropped from a height. Find the work done by its weight in one second after the ball is dropped.
10. A ball of mass 1 kg is dropped from a height of 5 m. (a) Find the kinetic energy of the ball just before it reaches the ground. (b) What is its speed at this instant?
11. A block is thrown upwards with a kinetic energy 1 J. If it goes up to a maximum height of 1 m, find the mass of the block.
12. A cycle together with its rider weighs 100 kg. How much work is needed to set it moving at 3 m/s?
13. A man performs 120 J of work in 6 s. Calculate the power.
14. Two persons do the same amount of work. The first person does it in 10 s and the second, in 20 s. Find the ratio of the power used by the first person to that by the second person.
15. How much work is done at the rate of 40 W in 40 s?
16. A 100-W electric bulb is used for five hours. How much energy is consumed by the bulb?
17. A 1000-W heater is used for 1 hour every day for 30 days. Find the cost of electricity if the rate is Rs 3.00/unit.

• ANSWERS •

A. ***Objective Questions***

I. **1.** (b) **2.** (b) **3.** (c) **4.** (a) **5.** (b)
6. (c) **7.** (c) **8.** (c) **9.** (d) **10.** (d)
11. (a) **12.** (b) **13.** (d) **14.** (b) **15.** (a)
16. (b)

II. **1.** F **2.** F **3.** F **4.** F **5.** T
6. T **7.** F **8.** T

E. ***Numerical Problems***

1. 196 J **2.** -9.8 J **3.** 60° **4.** 2.5 kg
5. 10 m/s **6.** 3.2 J **7.** 1/2 **8.** 39.2 J
9. 24 J **10.** (a) 49 J (b) $7\sqrt{2}$ m/s **11.** 102 g
12. 450 J **13.** 20 W **14.** 2 **15.** 1,600 J
16. 0.5 kWh **17.** Rs 90.00

• POSTSCRIPT •

- *Energy crisis*

We have learnt that energy can neither be created nor destroyed. Still we hear about the energy crisis and the need to save energy. In fact, energy comes in different forms but all forms do not suit our requirements. For example, there is lot of chemical energy in a chair made of wood, but you cannot run your fan with this chemical energy. For this we need electric energy. The crisis is for energy in usable forms. For that we have to depend on petroleum, coal, wind, sunlight, nuclear fuels like uranium, and so on. From these we get heat, electricity, etc., which can be used for our needs. Resources such as petroleum and coal are limited. Therefore, we have to use them judiciously.

Activities

- Take a toy car with a spring mechanism. Make a mark on a uniform floor with a chalk. Wind the key of the car through half a turn, and place the toy at the mark on the floor. Make another chalk mark at the point where the car stops. Measure the distance x between the two marks. This is the distance moved by the car. Rub out the second mark.

Now, wind the key through a full turn and place it at the first mark. Measure the distance the car moves before stopping. Repeat the experiment for one and a half turns of the key and two turns of the key. Measure the value of x in each case.

The greater the number of turns, the larger is the distance travelled by the car. This is because when the spring is compressed more, it stores greater energy. The distance x is roughly proportional to the energy stored in the spring. Try to get the relation between the energy stored and the number of turns of the key.

6 Sound

Hearing, sight, smell, taste and touch are the five senses we use to gather information about our surroundings. We hear when 'something' reaches our ear. What is that something? That something is sound.

NATURE OF SOUND

Sound is Produced by Vibrations

How is sound produced? Sound is produced when something vibrates. Sound is produced when we speak, when our fingers strike the membrane of a *tabla*, when signals are fed to the speakers of a radio, television or music system, when brakes are applied suddenly to stop a vehicle, when utensils bang against each other, and so on. In each case the source of sound vibrates. In many cases, you can feel this vibration by touching the source of sound. Hold a metal plate in one hand and hit it with a spoon. It will make a sound. If you touch the plate gently just after hitting it, you will feel vibrations.

Stretch a rubber band between your fingers and thumb. If you pluck it at the middle, the rubber band will start vibrating. If you bring the vibrating rubber band close to your ear, you will hear a sound.

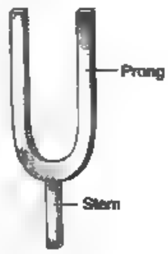


Fig. 6.1 Tuning fork

A tuning fork is used to produce a specific type of sound. It is a U-shaped steel device with a stem at the base. The tuning fork is held by the stem and one of its prongs is struck against a rubber pad. This causes the prongs to start vibrating, which produces a sound. Although the sound is clearly heard, it is not easy to see the vibrations of the prongs. However, you can confirm that the prongs are vibrating by doing the following activities.

1. Use tape to fix a string to a small plastic ball. Suspend the ball from a support. Now, strike a prong of a tuning fork against a pad, and touch the ball with the prong. The ball will move away with a jerk (Figure 6.2a). This shows that the prongs are vibrating. When they vibrate, they move back and forth. So, when a vibrating prong touches the ball, the ball moves.
2. Gently touch the surface of the water kept in a vessel with a prong of a tuning fork, after striking it against a pad. Since the prong vibrates, it creates ripples in water (Figure 6.2b).

We can now say that sound is produced by a vibrating body. In other words, the source of sound is a vibrating body. Let us now learn a bit more about the nature of sound.

Fig. 6.2

Sound Needs a Medium to Travel

Suspend an electric bell inside a bell jar by passing the connecting wires through an airtight cork fitted at the mouth of the jar. Place the jar over a disc which has a pipe connected to a vacuum pump.

When you press the switch of the bell, you will be able to hear it. Now, pump out the air from the jar with the help of the vacuum pump. Although it will not be possible to entirely remove the air from the jar, you should be able to remove most of it. Now, if you press the switch, you will be able to hear the bell feebly, if at all.

When there was air in the jar, sound travelled through it to the wall of the jar This caused the wall to vibrate and send the sound to you and everyone around it. But when air was removed, sound from the bell could not travel to the wall of the jar.

This shows that sound needs a medium to travel.

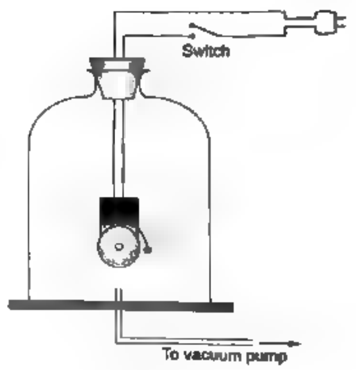


Fig. 6.3

Propagation of Sound

When a source of sound vibrates, it creates a 'disturbance' in the medium near it. This means that the condition of the medium near the source becomes different from its normal condition. The disturbance could be in the form of compression of the medium in an area near the source. This disturbance then travels in the medium. Let us see how.

Consider a vibrating membrane of a musical instrument like a drum or a tabla. As it moves back and forth, it produces a sound. Figure 6.4 shows the membrane at different instants and the condition of the air near it at those instants.

As the membrane moves forward (towards the right in Figure 6.4), it pushes the particles of air in the layer in front of it. So, the particles of air in this layer get closer to each other than normal. The density of air increases

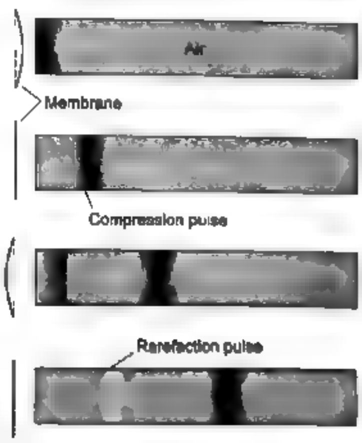


Fig. 6.4

locally, i.e., the air in that layer gets compressed. This layer of air pushes and compresses the layer next to it, which then compresses the next layer, and so on. In this way the disturbance moves forward. We call this type of disturbance a compression pulse. The particles of the medium do not travel with the compression pulse. Soon after pushing the particles in front, they return to their normal positions.

What happens when the membrane moves back (to the left)? It drags back the layer of air near it, decreasing the density of air there. The particles of air in the next layer on the right move in to fill this less dense (rarefied) area. As a result, its own density reduces. In the same way, the density of air in successive layers on the right decreases one after the other. We say that a rarefaction pulse moves to the right.

As the membrane moves back and forth repeatedly, compression and rarefaction pulses are produced, one after the other. These two types of pulses travel one behind the other, carrying the disturbance with it. This is how sound travels in a medium.

Variation in density and pressure in the medium during propagation of sound

When sound passes through a medium, we have alternate regions of high density and low density in the medium The regions of high density correspond to compressions, and the regions of low density correspond to rarefactions. Figure 6.5 shows an example of how the density of air varies when a sound wave passes through it. At places like *A, C, E* and *G* the density of air is maximum. At these places, the density is more than the normal density of air (when no sound is passing through it). At places like *B, D* and *F* the density is minimum, and it is less than the normal density of air.

Fig. 6.5

The density of air at any given place near a sound-producing body varies with time. In Figure 6.5, the density at *C* is maximum at the instant shown. After this if you continuously monitor the density at *C*, you will find that it gradually reduces to normal, and then reduces below the normal density till it reaches a minimum value. After this, the density will increase, go past the normal value and again become maximum. This variation in density is periodic, i.e., it is repeated after a fixed period of time.

A periodic variation in the value of a quantity between a maximum value and a minimum value is called oscillation.

Here, the change in density from the maximum value to the minimum value, and again to the maximum value makes one complete oscillation. In short, the occurrence of two consecutive maximum (or minimum) values makes one oscillation.

The pressure of air is related to its density and temperature. At a given temperature, the pressure of air is proportional to its density. As the density of air varies between a maximum and a minimum, so does the pressure. This variation in pressure on our eardrums enables us to hear sounds.

WAVE MOTION AND SOUND

We have seen that when a source of sound vibrates, compression and rarefaction pulses are produced one after the other These pulses travel one behind the other, carrying the disturbance with it. This kind of motion is called wave motion. In the case of sound, we say that a sound wave is travelling in the medium. Note that the particles of the medium do not travel with the disturbance. In the case of sound waves, the particles of the medium just move back and forth about their normal positions, creating compressions and rarefactions.

When a disturbance produced in one part of space travels to another part, without involving the transfer of any material with it, the motion of the disturbance is called wave motion. The disturbance itself is called a wave

One common example of waves is the ripples or circular waves on the surface of water, which are created when we drop a stone in a calm pool of water. Other examples of waves include sound waves, light waves, radio waves and TV waves.

You know that sound needs a medium to travel. But all waves do not need a material medium to travel. For example, light from the sun travels in space through vacuum before reaching the earth. Waves that do not need a material medium to propagate are called nonmechanical waves. Light, radio and TV waves are nonmechanical waves. Waves that need a material medium to propagate are called mechanical waves. Sound waves are mechanical waves.

Now let us learn a bit more about wave motion with the help of a slinky.

Waves on a Slinky

A slinky is a spring-shaped toy which can be extended or compressed very easily. It is very flexible and can be put into many shapes easily (Figure 6.6).

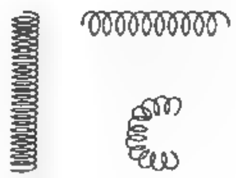


Fig. 6.6

Hang a slinky from a fixed support. Hold it gently at the lower end and quickly move your hand sideways and back. This will cause a hump on the slinky near the lower end. This hump will travel upwards on the slinky, as shown in Figure 6.7.

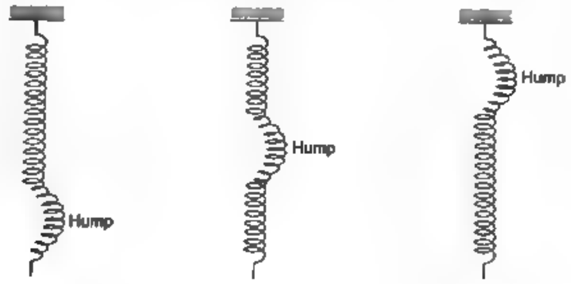


Fig. 6.7

What is travelling upwards? The part of the slinky that was at the bottom in the beginning is still at the bottom. Similarly, no other part of the slinky has moved up. Only the *disturbance* has moved up. Initially, the lower end of the slinky was disturbed. The disturbance then travelled upwards. Finally, the upper end of the slinky got disturbed. One may say that a wave has travelled up the slinky.

You can produce another kind of wave on a hanging slinky. Push its lower end up by about 5 cm and hold it there. This will compress the lower end of the slinky (Figure 6.8a). As time passes, you will see that the lower end will regain its normal shape, but the part just above it will become compressed (Figure 6.8b). So, the compression travels upwards. After some time, the part near the top will become compressed (Figure 6.8c).

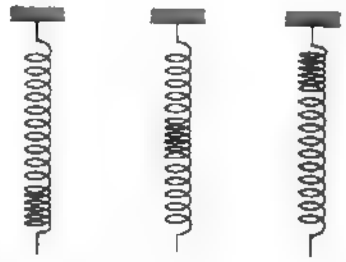


Fig. 6.8

Here also, a *disturbance* was created near the bottom of the slinky. The coils were forced to move a little upwards. This part of the slinky passed this disturbance to the part just above it. The coils there moved a little upwards, whereas the coils of the lower part regained their original positions.

Similarly, different parts of the slinky got disturbed at different times.

Notice that in this case, the coils on the slinky moved upwards when the disturbance reached them. The wave also moved upwards. But the motion of coils was very different from the motion of the wave. A coil moved only a small distance upwards, whereas the wave moved from the bottom to the top The part of the slinky which was near your hand initially, remained near your hand. Similarly for the other parts.

You can also send continuous waves on a slinky. Lay it down on a table or the floor, and ask a friend to hold one end. Pull the other end to stretch the slinky, and then move it to and fro along its length. You will see alternate compression and rarefaction of the coils (Figure 6.9). This is similar to the pattern of varying densities produced in a medium when sound passes through it

Fig. 6.9

Longitudinal and Transverse Waves

We have discussed two examples of waves travelling upwards on a slinky. In the first case (Figure 6 7), the coils of the slinky moved towards the right or left. So, they moved *perpendicular* to the direction of the wave motion. In the second case (Figure 6.8), the coils moved up and down, i.e., *along* the direction of the wave motion.

> If the particles of a medium move along the direction of motion of the wave, the wave is called a longitudinal wave
>
> If the particles of a medium move perpendicular to the direction of motion of the wave, the wave is called a transverse wave.

Look at figures 6.7 and 6.8. When a transverse wave passes through the slinky (Figure 6.7), its shape changes, and it becomes curved. On the other hand, when a longitudinal wave passes through it (Figure 6 8), its shape remains unchanged, but its density (number of coils per unit length) changes. These observations can be generalized as follows.

> Transverse waves involve change in the shape of the medium, whereas longitudinal waves involve change in the density of the medium

The shape of a liquid or a gas can be changed easily. This means that if the shape of a liquid or a gas is changed, it does not try to come back to its original shape. Hence, a transverse wave cannot be produced in a liquid or a gas. It can only be produced in a solid. On the other hand, every medium—solid, liquid or gas—tries to regain its original density if the density is changed by compressing or stretching (rarefying) it. Thus, longitudinal waves can be produced in all types of media, i.e., in solids, liquids and gases.

Sound waves are longitudinal

As we have seen, when a sound wave passes through a medium such as air, the layers in the medium are alternately pushed and pulled. The particles of the medium thus move to and fro along the direction of the propagation of the sound wave. Sound waves are therefore longitudinal.

Wavelength, Frequency and Wave Speed

Three quantities play an important role in describing the nature of a wave. These quantities are its wavelength, frequency and wave speed. They are called the *characteristic properties* of the wave. Here, we shall learn about them in the context of a sound wave.

Let us consider a sound wave produced by a source such as a tuning fork. Figure 6.10 shows the variation in the density of air near the source at a particular time as this wave passes through it. The variation in the density of air with distance is also shown by a graph in the figure. Since the pressure of air is proportional to its density, the plot of pressure versus distance will also have the same shape.

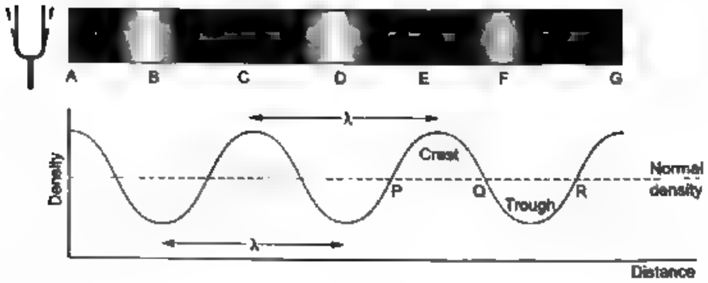


Fig. 6.10

It can be seen from the graph that in portions like PQ, the density is more than the normal density, and in portions like QR, it is less than normal. Portions where the density of the medium is more than the normal are called crests. Portions where the density of the medium is less than the normal are called troughs.

Wavelength

At any given instant, the density of air is different at different places along the direction in which sound is moving. For a source like a tuning fork, the distances between the consecutive positions of maximum density (like C and E in Figure 6.10) remain the same. The distances between the consecutive points of minimum density (like B and D) also remain the same. So, these values get repeated after a fixed distance. This distance is called the wavelength of the wave. It is denoted by the Greek letter λ (lambda). We can define wavelength as follows.

The minimum distance after which the state of disturbance is repeated in wave motion is called the wavelength of the wave.

Frequency

The density of air near a sound-producing body varies with time. Let us consider a sound wave produced by a source such as a tuning fork. Figure 6.11 shows the variation in the density of air at different instants, starting with time t_1. The pattern of alternate maximum and minimum density remains the same at all times. However, the pattern as a whole shifts forward as time passes.

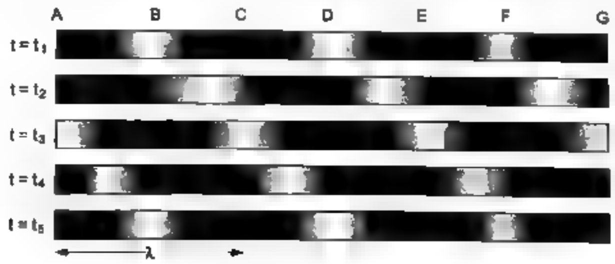


Fig. 6.11

Now, let us see what happens to the density at a particular position, say, C as time passes. The density there is maximum at t_1. After this, between t_1 and t_3, the density gradually reduces till it reaches the minimum value at t_3. Then, between t_3 and t_5, the density at C increases, till it again reaches the maximum value at t_5. So, in a period of time $t_5 - t_1$ the value of density changes from the maximum to the minimum, and then again to the maximum. In other words, it completes one oscillation. This time is called the time period (T) of the sound wave.

The time taken to complete one oscillation of the density of the medium is called the time period of the sound wave.

It follows that the maximum or minimum density of the medium at a place is repeated after an interval equal to the time period of the sound wave.

Frequency is a quantity that is closely related to time period. We can define the frequency of a sound wave as follows.

The number of oscillations of the density of the medium at a place per unit time is called the frequency of the sound wave

We usually use the Greek letter ν (nu) to denote frequency and T to denote time period. Frequency and time period are related as

$$\nu = \frac{1}{T} \quad \text{...6.1}$$

The unit of frequency is the hertz (Hz). Hertz is the same as s^{-1}, i.e., 1/second. If the density at a place in a medium makes 100 oscillations per second, the frequency is 100 Hz.

EXAMPLE 6.1 **A sound wave causes the density at a place in air to oscillate 600 times in 30 seconds. Find the time period and frequency of the sound wave.**

Solution The time period of the sound wave T = time taken to complete one oscillation

$$= \frac{30\ s}{600} = \frac{1}{20}\ s = 0.05\ s.$$

The frequency of the sound wave is $\nu = \frac{1}{T} = \frac{1}{0.05\ s} = 20$ Hz.

Compression (high density) and rarefaction (low density) are produced in a medium such as air due to the vibration of the source. So, the time period of the variation in density of the medium is the same as the time period of the vibration of the source. For example, a source vibrating with a time period of 0.01 s sends successive compression pulses at an interval of 0.01 s. At any given place, successive compression pulses will arrive at an interval of 0.01 s. So, the time period of the variation of density will be 0.01 s.

Clearly, the frequency of a sound wave is the same as the frequency of the vibration of its source.

Speed of sound

The speed with which a disturbance propagates in a medium is called wave speed.

Sound moves in a medium such as air with a finite speed. When your friend talks to you, it appears that you hear him immediately after he speaks. But that is not true. It takes some time for sound to reach you. This becomes clear when we see the flash of lightning first and hear the thunder after some time. Since light travels at a tremendous speed, we see the flash of lightning almost instantaneously Sound, travelling at a much lesser speed, takes more time to reach us.

The speed of a sound wave depends on the properties such as the temperature, pressure, etc., of the medium in which it is travelling. In air, the speed of sound is about 340 m/s. In water, sound travels at about 1,480 m/s.

Table 6.1 Speed of sound in different media

Medium	Temperature	Speed (m/s)
Air	0°C	332
	20°C	343
Nitrogen	0°C	970
Hydrogen	0°C	1284
Water	0°C	1402
	20°C	1482
Sea water	20°C	1520
Copper		4760
Stainless steel		5790

Relation between wavelength, frequency and wave speed

Look at Figure 6.11. The regions of high density correspond to compressions, and the regions of low density correspond to rarefactions. At time t_1, A is one of the places where the compression is the maximum. As time passes, this maximum compression shifts in the direction of the propagation of the wave. At time t_5, it reaches C.

Now, the distance AC = distance between consecutive maximum compressions
= wavelength, λ.

Also, the time $t_5 - t_1$ = time period, T.

So, the sound wave moves a distance λ in time T. Thus, the wave speed is

$$v = \frac{\lambda}{T}$$

or

$$v = \nu\lambda, \qquad \text{...6.2}$$

where ν is the frequency of the wave.

Sonic Boom

When a body moves with a speed which is greater than the speed of sound in air, it is said to be travelling at supersonic speed. Jet fighters, bullets, etc., often travel at supersonic speeds. And when they do, they produce a sharp, loud sound called a sonic boom.

The sonic boom produced by supersonic aircraft is accompanied by waves that have enough energy to shatter glass and even damage buildings. People living near the airbases of supersonic jets may also suffer hearing loss.

WHAT MAKES SOUNDS SEEM DIFFERENT TO US

We hear different kinds of sound. The sound of a sitar is quite different from that of a tabla. When we stand close to a loudspeaker, the sound produced by it seems very loud. But the same sound seems faint to a person standing a kilometre away. How sounds seem to us depends on many things. Let us see what they are.

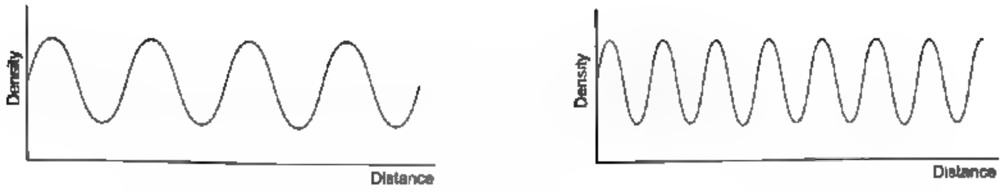
Pitch

We can easily distinguish between male and female voices. When someone plays the flute, we can distinguish between the different sounds produced when the different holes on the flute are closed. These sounds seem different to us because of their pitch. We say that a female voice has a higher pitch (is more shrill) than a male voice. The call of a koel has a higher pitch than that of a bull. The sounds of drums and tablas have a lower pitch than those of sitars and violins. Note that you cannot measure pitch and give it a value. You can only compare two sounds and say that one is at a higher pitch than the other.

The pitch of a sound depends on its frequency. The higher the frequency, the higher is the pitch. Our brain tells us whether a sound is of higher or lower pitch than another sound, even though we do not know the frequencies of the sounds. So, we can say the following.

Pitch describes how the brain interprets the frequency of sound.

Figure 6.12 shows the variations in density with distance for two sounds with different pitches. Note that the higher-pitch sound has a higher frequency as it completes more oscillations than the lower-pitch sound while covering the same distance.



Sound of lower pitch

Sound of higher pitch

Fig. 6.12

A tuning fork produces a sound of a particular frequency. But most of the other sounds that we hear do not have a unique frequency. For example, when we speak, the sound contains a mixture of frequencies. Its pitch is determined by the main frequencies in the sound.

Loudness

Suppose while watching TV, you find that the sound is hardly audible. You press a button on the TV's remote to increase the 'volume' of the sound, to make it louder. When you do that, what property of the sound wave produced by the TV set gets changed? The speed and frequency (hence the pitch) remain the same. What changes is the amplitude of the wave

Amplitude The amplitude of a sound wave in air can be described in terms of the density of air or the pressure of air or the displacement of the layers of air. You know that when sound travels in air, the layers of air move to and fro, causing compression and rarefaction. As a result, the density and pressure of air at a place vary. Their values increase from the normal to reach a maximum and then reduce to a minimum.

The amplitude of a sound wave in air is the maximum variation in density or pressure of air from the normal as the sound wave passes through

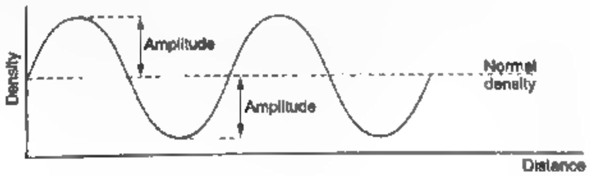


Fig. 6.13

We can also define amplitude as follows.

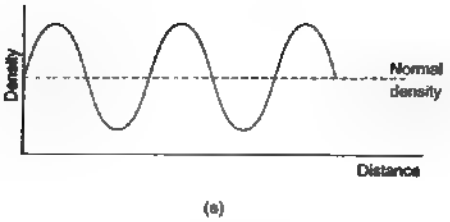
The amplitude of a sound wave in air is the maximum displacement of a layer of air from its mean position as the sound wave passes through.

The unit of amplitude depends on the quantity in terms of which the amplitude is being described. If it is in terms of density, the unit of amplitude is kg/m^3. If it is in terms of pressure, the unit is the pascal (N/m^2). And if it is in terms of displacement, the unit is the metre.

Amplitude and loudness

When sound reaches a layer of air, the layer starts vibrating. That means its energy increases. As the sound reaches another part of the air, the layer of air there starts vibrating, which shows that its energy has increased. This means that as sound wave travels, energy is transferred from one part of the medium to another. This energy depends on the amplitude of the wave. The larger the amplitude, the greater is the energy that passes through any place per unit time, and louder is the sound. So, the loudness of sound depends on the amplitude of the sound wave

Figure 6.14 shows the variations in density with distance for two sounds with different amplitudes. The amplitude in (a) is greater than the amplitude in (b). So, the graph in (a) represents a louder sound. The opposite of a loud sound is a soft sound. So, you can say that the graph in (b) represents a softer sound.



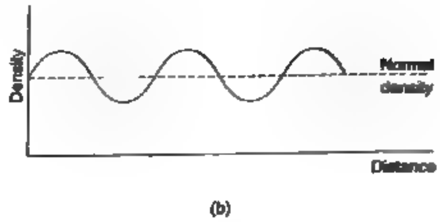


Fig. 6.14

As the sound wave travels, its initial energy gets spread over a progressively larger area. As a result, the energy of the layers of air decreases with distance, and they vibrate with lesser amplitude. So, the loudness of sound decreases with distance.

Quality of Musical Sound

A musical instrument produces sounds of different frequencies. The sound of a particular frequency is called a tone. For example, in Indian music, *sa, re, ga, ma, pa, dha* and *ni* represent tones. When we play a tone like *dha* on a musical instrument like the sitar, the sound that is produced has a mixture of frequencies. The loudest sound has the frequency (ν) of *dha*, and there are softer sounds whose frequencies are multiples of ν, like 2ν, 3ν, and so on. Such a sound seems more pleasant because it is fuller or richer than a sound of a single frequency. In general, a sound made up of different tones that have a frequency relation amongst them seems more pleasant. Such a sound is said to be of high quality.

RANGE OF HEARING

The human ear is able to hear sound in a frequency range of about 20 Hz to 20,000 Hz. As you know, the prefix kilo- is used to denote thousand. The above range of frequencies is thus written as 20 Hz–20 kHz. We cannot hear sounds of frequencies less than 20 Hz or more than 20 kHz. These limits vary from person to person and with age. Children can hear sounds of somewhat higher frequencies, say up to 30 kHz. With age, our ability to hear high-frequency sound diminishes. For the elderly, the upper limit often falls to 10–12 kHz. However, we take 20–20,000 Hz as the audible range for an average person.

Even in the audible range, the human ear is not equally sensitive to all frequencies. It is the most sensitive to frequencies around 2,000–3,000 Hz, where it can hear even a very low-intensity sound.

Sound of frequency less than 20 Hz is known as infrasonic sound or infrasound. Sound of frequency greater than 20 kHz is known as ultrasonic sound or ultrasound.

Different animals have different ranges of audible frequencies. A dog can hear sounds of frequencies up to about 50 kHz and a bat, up to about 100 kHz. Dolphins can hear sounds of even higher frequencies. These animals can also produce ultrasonic waves and communicate using them. Bats use ultrasonic waves to navigate, as we shall see a little later. Animals such as elephants and whales produce sounds of frequencies less than 20 Hz. Scientists have found that elephants grieve over their dead by producing infrasonic sounds. Some fishes can hear sounds of frequencies as low as 1–25 Hz. Rhinoceroses communicate using infrasonic sounds of frequency around 5 Hz.

It has been observed that animals behave in a peculiar manner before an earthquake or a volcanic eruption. It is believed that even before we actually see or feel the consequences of these events, sound waves are produced that are out of our audible range. For example, waves of frequencies less than 20 Hz reach the earth's surface before an earthquake. Animals and birds are able to detect these, and hence, start behaving in a different manner.

REFLECTION OF SOUND

Sound, like light, gets reflected at the surface of a solid or a liquid. Reflection of sound follows the same law as the reflection of light. When sound is reflected, the directions in which the sound is incident and reflected make equal angles with the normal to the reflecting surface, and the three are in the same plane. This can be verified as follows.

1. Take two long, identical tubes and place them on a table near a wall. Ask your friend to speak softly into one tube while you use the other tube to listen. You will find that you hear your friend's voice best when the tubes make equal angles with the wall, i.e., when $\angle i = \angle r$. Also, if you lift your tube off the table, you will not be able to hear your friend's voice clearly. This is because your tube, the incident sound and the normal are no longer in the same plane.
2. Repeat the experiment by placing flat objects of different materials (steel and plastic trays, a cardboard, a tray draped with cloth, etc.) against the wall. You will find that hard surfaces reflect sound better than soft ones.

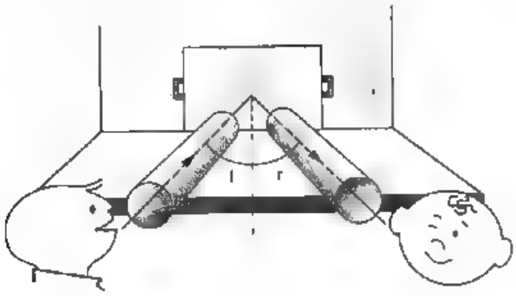


Fig. 6.15

As you have seen in the second part of the activity, the reflection of sound is dependent on the reflecting surface. Generally, hard surfaces reflect sound better than soft surfaces. But unlike light, which is reflected well only from highly polished surfaces, sound reflects quite well from rough surfaces as well. For example, an unplastered brick wall will reflect sound quite well. But even minor scratches on a mirror will affect reflection of light from it.

Multiple Reflections of Sound

Sound can get reflected a number of times before reaching us. The multiple reflections of sound has a number of uses and consequences. You might have heard the rolling of thunder, i.e., the prolonged sound of thunder that gradually fades away. This 'effect' is produced when the sound of thunder gets reflected from the surfaces of clouds and land multiple number of times, and the reflected sounds reach us one after the other. Let us now see how multiple reflections of sound is put to use by us.

1. In stethoscopes, the sound of a patient's heartbeat is guided along the tube of the stethoscope to the doctor's ears by multiple reflections of sound (Figure 6.16).
2. Horns, musical instruments such as trumpets and *shehanais*, loudhailers (megaphones), loudspeakers, etc., are all designed to keep sound from spreading in all directions. In these instruments, a tube followed by a conical opening reflects sound successively to guide most of the sound waves from the source in the forward

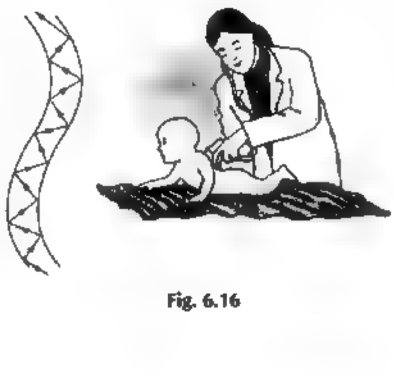


Fig. 6.16

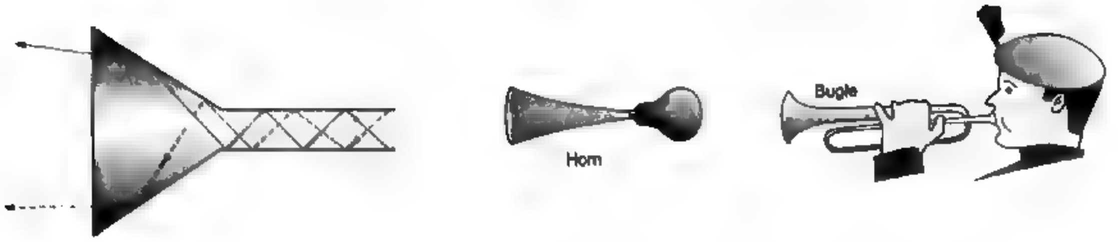


Fig. 6.17

direction, towards the audience (Figure 6.17). The sound waves add up and the loudness of sound increases.

3. Reflection of sound plays a big role in the design of concert halls. Designers have to keep the following things in mind: each member of the audience should hear the sounds on stage clearly and with equal loudness, and the sound should be pleasant.

   The ceiling of a concert hall can be curved so that sound, after reflection, reaches all corners of the hall (Figure 6.18a). A curved soundboard may be placed behind the stage so that sound after reflecting at the soundboard spreads evenly across the width of the hall (Figure 6.18b). The idea here, as you would have noticed, is similar to the use of parabolic reflectors in lamps. Apart from this, angled boxes are placed on the walls to reflect sound.

Reverberation Since a concert hall is designed to reflect sound from multiple surfaces, the sound produced on stage arrives at the listener through different paths (Figure 6.18a). Since the sound waves travel on different paths, they take different amounts of time to reach the listener. If the waves of the same sound arrive after a distinct interval, the listener will find it disturbing. However, the same sound arriving in quick succession will appear to fade away in a pleasing way, i.e., the sound will appear rich and pleasant.

The persistence of sound due to repeated reflection and its gradual fading away is called reverberation of sound

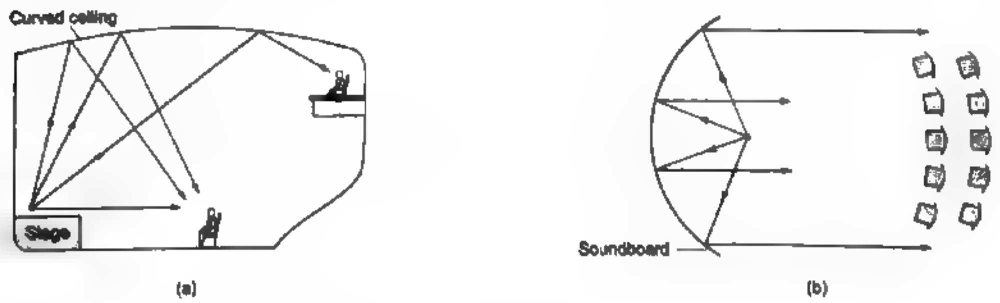


Fig. 6.18

If there are too many repeated reflections, the reflected sound mixes with the next sound coming from the stage, which is not desirable. So, certain materials which absorb sound are placed judiciously around the hall to prevent unwanted reflection from those areas. But, soft furnishing is avoided in concert halls because they absorb sound.

Echo

If you stand in front of a hill and light a firecracker, you will hear its sound twice. First you will hear the sound coming directly from the firecracker, and then you will hear its sound reflected off the hill. The sound that is heard after the first sound, due to reflection of sound at a surface, is called echo. You must have heard your voice echoed in large, empty halls also.

There are buildings whose construction causes echoes to be heard clearly and loudly. For example, if you stand at the centre of a hemispherical hall and clap, the sound will start in all directions, reflect from the walls and return to the centre, which acts as the focus for the reflected sound. So, you will clearly hear the echo of the clap. You can also hear echoes in buildings with large domes.

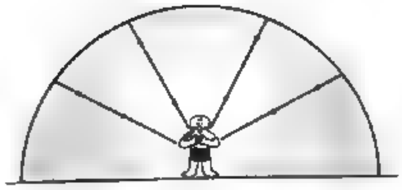


Fig. 6.19

Sometimes echoes can be disturbing. Often, when a speaker addresses a public meeting in a large ground, echoes from nearby buildings make it difficult to understand what the speaker is saying.

When we talk in a room, why do we not hear echoes due to reflections from the walls and the ceiling? This is because the human ear cannot distinguish between two sounds if they arrive at an interval of less than about one-fifteenth of a second. This time varies from person to person and also with the frequency of the sound waves. Usually it is between ($\frac{1}{10}$) to ($\frac{1}{20}$) s. We will use ($\frac{1}{15}$) s as a typical interval needed to distinguish two sounds. In a typical room the walls are only a few metres away from the speaker. Sound travels in air at about 340 m/s. Therefore, it takes about one-fiftieth of a second or less for the reflected sounds to arrive after the direct sound. The reflected sounds are not detected as separate sounds from the first one. As a matter of fact, they add to the loudness of the first sound, and as with concert halls, it makes the sound more pleasant.

Since sound travels at about 340 m/s in air, for the reflected sound to be audible separately, sound must travel at least 340 m/s × ($\frac{1}{15}$) s = 22.67 m. Therefore, you will hear the echo of your voice only when the nearest reflecting surface is at least (22/2) m, or about 11 m away.

Knowing the speed of sound in a medium such as air or water, and the time between a sound and its echo, we can calculate the distance of a reflecting surface. For example, let the time between hearing the direct sound and its echo off a hill be 2 seconds.

Total distance travelled by sound in going to the hill and back = speed × time = (340 m/s × 2 s).

$$\therefore \quad \text{the distance of the hill from the sound source} = \frac{1}{2} \times (340 \times 2)\ \text{m} = 340\ \text{m}.$$

Use of echos by bats and other animals

Nature has equipped animals such as bats and porpoises with a mechanism by which they find their distance from an object using echos. They emit ultrasonic squeaks which bounce off surfaces in front of them and return as echoes. From the time between a squeak and its echo, they can judge the distance of an obstacle or a prey in front of them. They can also judge whether the thing in front of them is an obstacle or a prey, whether it is moving or not, its size, etc. Thus, these animals use sound to 'see'. Man has also learnt to use this method to 'see' indirectly, as we shall learn shortly.

Fig. 6.20

ULTRASOUND AND ITS APPLICATION

A compression-and-rarefaction wave in a material medium with frequencies above 20 kHz is known as an ultrasonic wave or ultrasound. Such waves can travel quite freely in solids and liquids. However, their intensity drops drastically in gases. Because of their high frequencies, ultrasonic waves have some special properties, which audible sound does not have. Let us now study two properties of ultrasonic waves that make them very useful to us.

Properties of Ultrasonic Waves

High power

A wave carries energy with it. This energy may be transferred to the medium through which the wave is passing. The energy of a wave depends on several factors, including its frequency. Due to its high frequency, an ultrasonic wave can be made very powerful. When you shout at your loudest, you produce energy of about 0.001 J per second, i.e., a power of about 0.001 W. However, if a source vibrates with the same amplitude as the sound you produced, but at 1,000 kHz, it would produce 1,000 W of power! The high power of ultrasonic waves is used in industry and medicine to break and cut objects.

Good directionality

Ultrasonic waves are able to travel along well-defined straight paths, even in the presence of obstacles. This means they do not bend appreciably at the edges of the obstacle. When a low-frequency sound meets an obstacle, it easily bends round the corners and spreads in all directions. If you put a board between your face and a source of sound, you are still able to hear the sound. This is because sound waves bend at the corners of the board and reach your ears. At the high frequencies of ultrasound, this bending phenomenon is quite insignificant, and the waves continue to move along straight lines. In this respect ultrasonic waves are closer to light. Therefore, they are used for imaging objects. The benefit over light is that ultrasonic waves can travel in nontransparent materials also, where light cannot be sent.

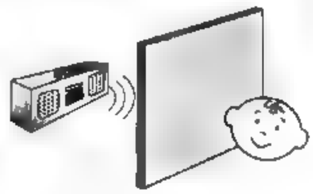


Fig. 6.21

Industrial Applications of Ultrasonic Waves

Ultrasounds are used extensively in industry. Here are a few examples.

Drilling holes or making cuts of desired shape

You can use a hammer and a steel punch to make holes in metal plates, plastic sheets or other solid materials. Such holes can also be made using ultrasonic vibrations produced in a metallic rod, called a *horn*. The horn acts like a hammer, hammering the plate about a hundred thousand times per second. The shape of the hole is the same as that of the tip of the horn. The shape of the tip can be designed as per the requirement of the application. Ultrasonic cutting and drilling are very effective for fragile materials like glass, for which ordinary methods might not succeed.

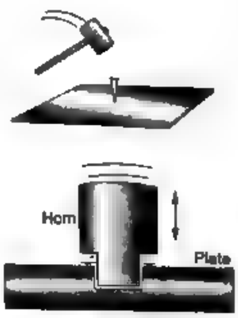


Fig. 6.22

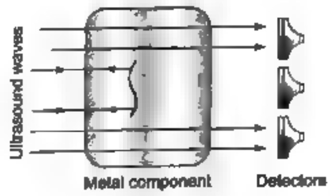
Ultrasonic cleaning

We normally clean dirty clothes, plates or other large objects by dipping in a detergent solution, and then rubbing and washing. But for small parts such as those used in watches, electronic components, odd-shaped parts such as a spiral tube, and parts located in hard-to-reach places, this method is inconvenient and sometimes impossible. Such objects are placed in a cleaning solution and ultrasonic waves are sent into the solution. This causes high-frequency vibrations in the solution. This knocks off all dirt and grease particles from the objects.

Ultrasonic detection of defects in metals

Metallic components are used in buildings, bridges, machines, scientific equipment, and so on. If there are cracks or holes inside the metal used, the strength of the structure or component is reduced and it can fail. Such defects are not visible from the outside. Ultrasonic waves can be used to detect such defects.

Ultrasonic waves are sent through the metallic object under study. If there is no crack or cavity in its path, it goes through the object. A detector placed on the other side detects the transmitted wave. A defect present in the path of the wave reflects the wave. Thus, the intensity of the emerging waves falls in the region that is in line with the defect. When this happens, we know that the object has a defect inside.



6.23

Why cannot ordinary sound be used for this application? This is because ordinary sound will bend considerably round the corners of cracks or cavities, and will emerge on the other side at almost full intensity.

Medical Applications of Ultrasound

Imaging of organs

Ultrasonic waves have given doctors powerful and safe tools for imaging human organs. Echocardiography is a technique in which ultrasonic waves, reflected from various parts of the heart, form an image of the heart. Ultrasonography is routinely used to show doctors images of a patient's organs such as the liver, gall bladder, uterus, etc. It helps doctors detect abnormalities such as stones in the gall bladder, tumours, etc. It is also used to monitor the growth of a foetus inside the mother's womb


Fig. 6.24 Ultrasound image showing twins (F1 and F2)

These applications are based on the high directionality of ultrasound waves and their capability to reflect from the boundaries between different kinds of material. Ultrasonic waves of low intensity are sent to the desired area of the body The waves travel along straight lines till they hit an internal structure. A part of the wave is reflected from here, and the rest is transmitted to the next structure. It is again reflected at the next boundary, and so on. Waves are sent from different angles, and all the reflected waves are gathered by a receiver. These waves are then converted into electrical signals that are used to generate images of an organ. These images are then displayed on a monitor, and if required printed on film.

Ultrasonography is safer than the older X-ray imaging technique. Repeated X-rays can harm tissues, especially those of a foetus.

Surgical use of ultrasound

The ability of ultrasonic waves to cause molecules of materials to vibrate vigorously, and thus causing certain materials to break into tiny pieces (emulsify) is employed in ultrasound surgery. Cataract removal is a very common example. The lens of the eye is made up of a clear gel surrounded by a firm capsule. With old age, this gel may become cloudy and hard. Light passing through it becomes diffused, causing blurry vision. A technique called phakoemulsification

(*phako* stands for the lens of the eye) uses the power of ultrasound to break the hardened gel into tiny pieces.

The surgeon makes a small cut (about 3 mm) in the lens capsule and inserts an ultrasound probe. Ultrasonic waves of, say, 40 kHz are sent through the probe, breaking the hardened gel into tiny pieces. These pieces are then sucked out. A plastic lens is placed in the capsule, restoring vision.

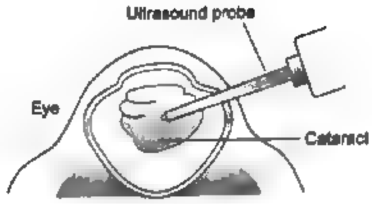


Fig. 6.25

Ultrasound is also employed to break small 'stones' that form in the kidneys into fine grains. These grains get flushed out with urine. This method has eliminated the need to perform surgery.

Sonar

Sonar stands for sonographic navigation ranging. This is a method for detecting and finding the distance of objects under water by means of reflected ultrasonic waves. The device used in this method is also called sonar.

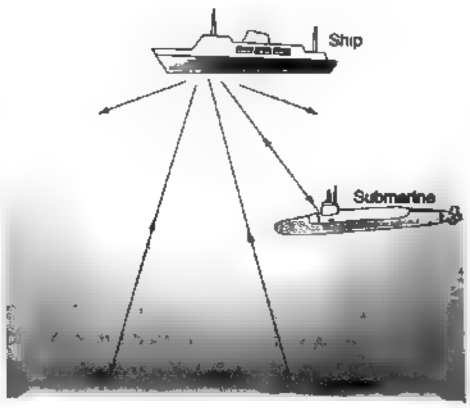


Fig. 6.26

From the observation centre on board a ship, ultrasonic waves of high frequencies, say 1,000 kHz, are sent in all directions under the water. These waves travel in straight lines till they hit an object such as a submarine, a sunken ship, a school of fish, etc. The waves are then reflected, and are received back at the observation centre. The direction from which a reflected wave comes to the observation centre tells the direction in which the object is located. From the time between sending the ultrasonic wave and receiving its echo, and the speed of sound in sea water, the distance of the object from the observation centre is calculated. Reflections from various angles can be ultilized to determine the shape and size of the object.

Let d = distance between the sonar and an underwater object,

t = time between sending an ultrasonic wave and receiving its echo from the object, and

v = speed of sound in water.

The total distance covered by the wave from the sonar to the object and back is $2d$.

Using $s = ut$,

$$2d = vt$$

or

$$d = \frac{vt}{2}.$$

This method of finding distances is also called echo ranging. Marine geologists use this method to determine the depth of the sea and to locate underwater hills and valleys.

THE HUMAN EAR

Our ears are one of our most important sense organs. Many principles of physics are involved in its working Figure 6.27 shows the main parts of the human ear. The structure of the human ear can be divided into three parts—the outer ear, the middle ear and the inner ear.

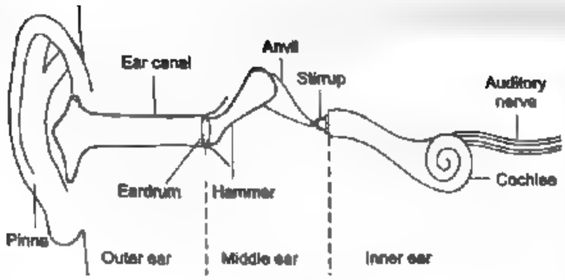


Fig. 6.27

Outer ear The outer ear consists of the pinna and the ear canal. Pinna is the part which is visible from the outside. Also, visible is the opening of the ear canal, which is about 2 cm long. The function of the outer ear is to guide sound waves to the middle ear.

Middle ear The middle ear is separated from the ear canal by a tightly stretched membrane called the eardrum or tympanic membrane. Beyond it lie three interconnected bones called the hammer, anvil and stirrup The function of the middle ear is to pick up, amplify and transmit sound waves to the inner ear.

Inner ear The inner ear consists of a liquid-filled coiled tube called the cochlea, which is shaped like a snail. This tube is connected to the stirrup. The cochlea has special sensory cells called hair cells, which are so called because of the hairlike structures that stick out of them. The hair cells are connected to the auditory nerve, which is connected to the brain.

Working of the Human Ear

You know that the vibrating source of a sound sets up compressions and rarefactions in air. When these reach the outer ear, they are guided into the ear canal. As the air near the eardrum is compressed, it pushes the eardrum inwards. And as it gets rarefied, the eardrum moves outwards. In this way the eardrum starts vibrating.

A vibration of the eardrum makes each of the interconnected bones—the hammer, anvil and stirrup—to vibrate. These interconnected bones act like levers, which increase the displacement during movement or vibration. So, the displacement of the stirrup becomes many times that of the eardrum. In this way, the amplitude of vibrations gets amplified as the vibrations reach the stirrup. The stirrup, which is connected to the cochlea, sets up compression and rarefaction pulses in the fluid in the cochlea. These pulses cause the hairlike structures in the cochlea to move. This creates electrical signals, which are sent to the brain through the auditory nerve. The brain interprets these signals to give us the sensation of hearing sound.

SOLVED PROBLEMS

A sound-wave source produces 20 crests and 20 troughs in 0.2 second. Find the frequency of the wave.

Solution Since the sound wave produces 20 crests in 0.2 second, the density of the medium at a place becomes maximum 20 times in 0.2 second. The number of times it becomes maximum in 1 second is

$$\frac{20}{0.2} = 100.$$

So, the frequency is 100 times/second or 100 Hz.

A person is sitting 200 metres away from a 20-Hz sound source. What is the time interval at which successive compression pulses from the source reach him?

Solution We have, $\nu = 20$ Hz.

Time period of the sound wave,

$$T = \frac{1}{\nu} = \frac{1}{20\ \text{Hz}} = 0.05\ \text{s}.$$

This is the time interval between successive compression (or rarefaction) pulses,

∴ successive compression pulses reach the person after 0.05 s.

EXAMPLE 3 **Calculate the wavelength of a sound wave whose frequency is 300 Hz and speed is 330 m/s.**

Solution We have, $v = \nu\lambda$

or $$\lambda = \frac{v}{\nu} = \frac{330\ \text{m/s}}{300\ \text{s}^{-1}} = 1.1\ \text{m}.$$

EXAMPLE 4 **A sound wave has a frequency of 1,000 Hz and a wavelength of 34 cm. How long will it take to travel 1 km?**

Solution We have, $\nu = 1000$ Hz and $\lambda = 34\ \text{cm} = \frac{34}{100}$ m.

The wave speed is $v = \nu\lambda = (1000\ \text{s}^{-1})\left(\frac{34}{100}\text{m}\right) = 340$ m/s.

The time taken by the wave to travel 1 km is

$$t = \frac{s}{v} = \frac{1\ \text{km}}{340\ \text{m/s}} = \frac{1000\ \text{m}}{340\ \text{m/s}} = 2.94\ \text{s}.$$

EXAMPLE 5 **A dog barks in a park and hears its echo after 0.5 second. If the sound of its bark got reflected by a nearby building, find the distance between the dog and the building. Take the speed of sound in air as 346 m/s.**

Solution Let the distance of the dog from the building $= d$.

The distance travelled by sound in 0.5 s is $2d$.

From $s = ut$,

$2d = 346\ \text{m/s} \times 0.5\ \text{s}$

∴ $$d = \frac{346\ \text{m/s} \times 0.5\ \text{s}}{2} = 86.5\ \text{m}.$$

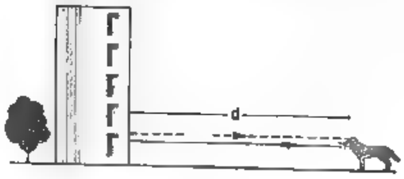


Fig. 6.W1

EXAMPLE 6 **The deepest recorded point below the sea level is in the Mariana Trench, Pacific Ocean. This point is 11 km below the sea level. A research vessel sends down a sonar signal to confirm this depth. After how long can it expect to get the echo? Take the speed of sound in sea water as 1,520 m/s.**

Solution The total distance travelled by the sonar signal from the ship to the sea bed and back

$= 2 \times 11\ \text{km} = 22{,}000$ m.

Time taken by sound to travel this distance,

$$t = \frac{s}{v} = \frac{22000\ \text{m}}{1520\ \text{m/s}} = 14.47\ \text{s}.$$

∴ the echo will reach the ship 14.47 seconds after the signal is sent.

EXAMPLE 7 **A construction worker's helmet slips and falls when he is 78.4 m above the ground. He hears the sound of the helmet hitting the ground 4.23 seconds after it slipped. Find the speed of sound in air.**

Solution Let t be the time taken by the helmet to reach the ground.

We have, $h = ut + \frac{1}{2}gt^2$.

Here $u = 0$, $h = 78.4$ m, $g = 9.8\ \text{m/s}^2$.

∴ $78.4\ \text{m} = \frac{1}{2} \times (9.8\ \text{m/s}^2) \times t^2$

or $t^2 = \frac{2 \times 78.4\ \text{m}}{9.8\ \text{m/s}^2}$ or $t = 4$ seconds.

∴ the time taken by the helmet to drop to the ground = 4 s.

The time taken by sound to travel 78.4 m = (4.23 − 4) s = 0.23 s.

∴ the speed of sound in air = $\frac{78.4 \text{ m}}{0.23 \text{ s}}$ = 340.87 m/s ≈ 341 m/s.

• POINTS TO REMEMBER •

- *Wave motion and the nature of sound*

  When a disturbance produced in one part of space travels to another part, without involving the transfer of any material with it, the motion of the disturbance is called *wave motion*. The disturbance itself is called a *wave*.

  If the particles of a medium move along the direction of the motion of the wave, the wave is called a *longitudinal wave*. Sound waves are longitudinal.

  If the particles of a medium move perpendicular to the direction of the motion of the wave, the wave is called a *transverse wave*

  Sound is a wave motion, produced by a vibrating source. A medium is necessary for its propagation as it is a mechanical wave.

  A vibrating source produces compression and rarefaction pulses, one after the other in the medium. These pulses travel one behind the other as a sound wave.

  Compression pulses correspond to regions of high density and pressure, whereas rarefaction pulses correspond to regions of low density and pressure in the medium

  The minimum distance after which the state of disturbance is repeated in a wave motion is called the *wavelength* (λ) of the wave.

  The time taken to complete one oscillation of the density of the medium is called the *time period* (T) of the sound wave.

  The number of oscillations of the density of the medium at a place per unit time is called the *frequency* (ν) *of the sound wave.*

  $$\nu = \frac{1}{T}.$$

  The speed, frequency and wavelength of a sound wave are related by the equation

  $$v = \nu\lambda.$$

- *Sounds of different frequencies*

  An average person can hear sounds in the frequency range 20 Hz–20 kHz Sounds below 20 Hz are called *infrasound* or *infrasonic sound*, and those above 20 kHz are called *ultrasound* or *ultrasonic sound*.

- *Reflection of sound*

  When sound is reflected, the directions in which the sound is incident and reflected make equal angles with the normal to the reflecting surface and the three are in the same plane.

  Hard surfaces reflect sound better than soft ones.

  Reflection of sound has many applications such as in stethoscope, horns, megaphones, sonar ultrasonography, etc.

  If a sound and its reflection from a surface arrive at an interval of one-fifteenth of a second or more, we hear an *echo*.

  The persistence of sound due to repeated reflections and its gradual fading away is called *reverberation of sound*.

- *Ultrasound and its applications*

  Two properties of ultrasound make it useful to us—its high power and the fact that it does not bend appreciably around obstacles.

  Some industrial applications: Cutting and drilling holes in fragile materials; cleaning parts or areas which are small, odd-shaped, hard to reach, etc, fault-detection in metallic components and structures.

  Some medical applications: imaging of organs (ultrasonography, echocardiography), in cataract removal, in breaking stones in the kidney

  Sonar is a method for detecting and finding the distance of objects under water by getting ultrasonic waves to reflect off them.

• EXERCISES •

A. *Objective Questions*

I. *Pick the correct option.*

1. When we say 'sound travels in a medium', we mean
   (a) the particles of the medium travel
   (b) the source travels
   (c) the disturbance travels
   (d) the medium travels

2. A sound wave consists of
   (a) a number of compression pulses one after the other

(b) a number of rarefaction pulses one after the other
(c) compression and rarefaction pulses one after the other
(d) a compression and a rarefaction pulse separated by a distance equal to one wavelength

3. The time period of a sound wave travelling in a medium is T. At a given instance ($t = 0$) a particular region in the medium has minimum density. The density of this region will be minimum again at
(a) $t = T$ (b) $t = T/2$
(c) $t = T/3$ (d) $t = T/4$

4. The frequency, wavelength and speed of a sound wave are related as
(a) $\nu = v\lambda$ (b) $\lambda = v\nu$
(c) $\nu = \lambda/v$ (d) $v = \nu\lambda$

5. Hertz stands for
(a) second (b) second $^{-1}$
(c) metre (d) metre $^{-1}$

6. The frequency of a source is 20 kHz. The frequencies of the sound waves produced by it in water and air will
(a) be the same as that of the source
(b) depend upon the velocity of the waves in these media
(c) depend upon the wavelength of the waves in these media
(d) depend upon the density of the media

7. If the density of air at a point through which a sound wave is passing is maximum at an instant, the pressure at that point will be
(a) minimum
(b) same as the density of air
(c) equal to the atmospheric pressure
(d) maximum

8. When we increase the loudness of sound produced by a radio, the property of the sound wave that changes is its
(a) amplitude (b) frequency
(c) speed (d) wavelength

9. The term that describes how the brain interprets the frequency of a sound is called
(a) amplitude (b) frequency
(c) pitch (d) pinna

10. An object moving at a speed greater than that of sound is said to be moving at
(a) ultrasonic speed (b) sonic speed
(c) infrasonic speed (d) supersonic speed

11. In a stethoscope, sound of heartbeats travel through the stethoscope's tube
(a) by bending along the tube
(b) in a straight line
(c) by undergoing multiple reflections
(d) as a sonic boom

12. The properties of ultrasound that make it useful are
(a) high power and high speed
(b) good directionality and high power
(c) high speed and frequency
(d) good directionality and ability to move around objects

13. Ultrasonic waves are used for detecting objects under water. What technique/device is used for this?
(a) Ultrasonography (b) Echocardiography
(c) Phakoemulsification (d) Sonar

14. The eardrum is a
(a) bone (b) coiled tube
(c) stretched membrane (d) fluid

15. The part of the ear that is filled with a liquid is the
(a) cochlea (b) ear canal
(c) anvil (d) hammer

II. *Fill in the blanks*

1. Sound is produced by a ...... body.
2. The relation between the frequency (ν) and time period (T) of a sound wave is ....
3. A medium carrying a sound wave receives compression pulses every 0.01 s. The time period of the wave is ...... and the frequency of the source is ....
4. A sound of frequency less than 20 Hz is called ...... sound.
5. The loud, sharp sound heard when a supersonic aircraft flies overhead is called ....
6. The property of ultrasound used to detect flaws in metals is its good ....
7. The cochlea has sensory cells called ....
8. In the human ear, the hammer is connected to the eardrum on one side and to the ...... on the other.

B. *Very-Short-Answer Questions*

Answer the following in one word or maximum one sentence.

1. Is sound wave longitudinal or transverse?
2. Name two quantities that vary periodically at a place in air as a sound wave travels through it.
3. The frequency of a source of sound is 10 Hz. How many times does it vibrate in a minute?
4. In which of the three media—air, water or steel—does sound travel the fastest?
5. Which has a higher pitch—the sound of a whistle or that of a drum?
6. Which has higher frequency—infrasonic sound or ultrasonic sound?
7. The grandparents and parents of a two-year-old girl are playing with her in a room. A sound source produces a 28-kHz sound. Who in the room is most likely to hear the sound?
8. Does sound follow the same laws of reflection as light does?
9. Why is soft furnishing avoided in concert halls?
10. Name two surgical applications of ultrasound.
11. Two sources A and B vibrate with the same amplitude. They produce sounds of frequencies 1 kHz and 30 kHz respectively. Which of the two waves will have larger power?

12. Which part of the ear sets up pulses in the fluid in the cochlea?

C. *Short-Answer Questions*

Answer the following in 30–40 words each

1. Define wave motion.
2. Define longitudinal and transverse waves.
3. What is the difference between a mechanical and a nonmechanical wave? Give one example of each.
4. Define the terms 'crest' and 'trough' in a wave.
5. What do you understand by a sound wave?
6. Why do we say that sound waves are longitudinal?
7. Define the wavelength of a sound wave. How is it related to the frequency and wave speed?
8. What do you understand by a sonic boom?
9. What do you understand by range of hearing in humans? Around which frequency of sound are our ears most sensitive?
10. Give an example to show that sound travels at a finite speed.
11. Explain how echoes are used by bats to locate obstacles and prey in front of them.
12. State the special properties of ultrasound that make it useful to us. In general, how are these properties utilized?
13. How are small parts cleaned using sound waves?
14. Why does sound become faint with distance?
15. Write the structure of the middle ear.
16. How do the vibrations of the eardrum get transmitted to the fluid in the cochlea?

D. *Long-Answer Questions*

Answer the following in not more than 70 words.

1. Define the wavelength, time period and frequency of a sound wave.
2. Sound cannot travel in vacuum. Describe an experiment to demonstrate this.
3. With the help of a diagram describe how compression and rarefaction pulses are produced in air near a source of sound.
4. Figure 6E.1 shows the density at an instant at various places ($A, B, C, \ldots$) in air as a sound wave travels through it. Now answer these questions:
   (a) At B, what would you find—a compression or a rarefaction pulse?
   (b) Name three places where the pressure is below the atmospheric pressure.
   (c) Explain wavelength with reference to the figure.
   (d) Mark off two distances in the figure to show wavelength, keeping in mind your explanation.

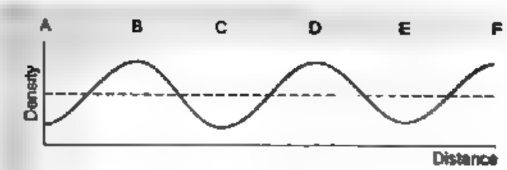


Fig. 6.E1

5. How do echoes in a normal room affect the quality of the sounds we hear?
6. Explain briefly how a flaw in a metal component can be detected using ultrasound.
7. What is the role of the ultrasonic probe in cataract surgery?
8. Explain the working and applications of sonar.
9. Explain the sturcture of the human ear with the help of a diagram.

E. *Numerical Problems*

1. Find the frequency of a wave whose time period is 0.002 second.
2. Find the time period of the source of a sound wave whose frequency is 400 Hz.
3. A sound wave causes the density of air at a place to oscillate 1200 times in 2 minutes. Find the time period and frequency of the wave.
4. A source produces 15 crests and 15 troughs in 3 seconds. When the second crest is produced, the first is 2 cm away from the source. Calculate (a) the wavelength, (b) the frequency, and (c) the wave speed.
5. A sound wave travels at a speed of 340 m/s. If its wavelength is 2 cm, what is the frequency of the wave? Will it be in the audible range?
6. Given that sound travels in air at 340 m/s, find the wavelength of the waves in air produced by a 20-kHz sound source. If the same source is put in a water tank, what would be the wavelength of the sound waves in water? Speed of sound in water = 1,480 m/s.
7. A child watching Dusserah celebrations from a distance sees the effigy of Ravana burst into flames and hears the explosion associated with it 2 s after that. How far was he from the effigy if the speed of sound in air that night was 335 m/s?
8. The war correspondent of a TV channel watches the flashes of two tanks firing at each other at the same time on a straight road ahead of him. He hears the sounds of the two shots 2 seconds and 3.5 seconds after seeing the flashes. Taking the speed of sound in air as 340 m/s, he quickly calculates and reports the distance between the tanks. What distance did he report?

9. A bat in search of prey hears the echo of its squeak after 0.1 s. How far is the prey from the bat, if the squeak got reflected from it? (Speed of sound in air = 340 m/s.)
10. A fishing boat using sonar detects a school of fish 190 metres below it. How much time elapsed between sending the ultrasonic signal which detected the fish and receiving the signal's echo? (Speed of sound in sea water = 1,520 m/s.)
11. A monkey drops a coconut from the top of a tree. He hears the sound of the coconut hitting the ground 2.057 seconds after dropping it. If the monkey was 19.6 metres above the ground, what is the speed of sound in air? (Take $g = 9.8\ m/s^2$.)

✦

· ANSWERS ·

A. *Objective Questions*

I. 1. (c) 2. (c) 3. (a) 4. (d) 5. (b)
6. (a) 7. (d) 8. (a) 9. (c) 10. (d)
11. (c) 12. (b) 13. (d) 14. (c) 15. (a)

E. *Numerical Problems*

1. 500 Hz 2. 0.0025 s
3. 0.1 s, 10 Hz 4. (a) 2 cm (b) 5 Hz (c) 10 cm/s
5. 17,000 Hz, Yes 6. 1.7 cm, 7.4 cm 7. 670 m
8. 510 m 9. 17 m 10. 0.25 s 11. 344 m/s

Question Bank

Includes questions that require higher-order thinking skills (HOTS)

Very-Short-Answer Questions

Measurements

1. Is hatred a physical quantity?
2. Write four things that are not physical quantities.
3. Which unit is longer—inch or centimetre?
4. The perimeter of a circle is $2\pi r$, where r is the radius of the circle. Is perimeter a base quantity or a derived quantity?

Describing Motion

5. Of the speeds 20 km/h and 5 m/s, which is greater?
6. Of the speeds 32 km/h and 10 m/s, which is greater?
7. What will be the SI unit of velocity/acceleration?

Force and Acceleration

8. What is the direction of the resultant force on a particle that moves with a uniform speed in a circular path of radius R?

Gravitation

9. If the time required by a planet to go round the sun is denoted by T and its mean distance from the sun is denoted by D then $T = D^n$. What is the value of n?

Work, Energy and Power

10. A crow is flying horizontally. What is the work done on it by the force of gravity?
11. You are walking against a strong wind. Is the work done by the wind on you positive, negative or zero?
12. A force acts on an object northwards, and the object moves in the south-west direction. Is the work done by the force zero, negative or positive?
13. Raindrops are falling on the concrete roof of a building. Is there work done by the raindrops?
14. Is energy a vector quantity or a scalar quantity?

Short-Answer Questions

Measurements

1. Write the names of ten physical quantities.
2. Can the distance covered by you while taking a step be a good unit of measurement? Give reasons.
3. Someone asks, "Which unit is larger—year or light-year?" What will you say?
4. Why is joule considered a derived SI unit?

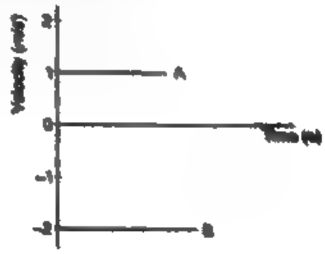


Fig. QB-1

Describing Motion

5. The velocity–time plots of two objects A and B are shown in the figure.
   (a) Which one is moving faster?
   (b) Does the separation between the two increase, decrease or remain constant as time passes?

6. It is said that the area under a velocity–time graph gives displacement. Comment on this statement in view of the fact that area is measured in square metres and displacement, in metres.

Force and Acceleration

7. What minimum number of forces is needed to deform a body without moving it?
8. What is the weight of a 1-kg body?
9. The symbol for the SI unit of force is the same as the chemical symbol of an element. Name the element.
10. Write the unit 'newton' in terms of kilograms, metres and seconds.
11. An object of mass m moves with a uniform speed v inside a plastic pipe bent at one end. The bent portion forms a circular arc of radius R. What is the resultant force on the object when it moves through (a) the straight portion of the pipe, and (b) the bent portion of the pipe?

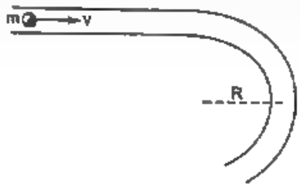


Fig. QB-2

Work, Energy and Power

12. Give two examples, where the work done by a force is negative.

Long-Answer Questions

Describing Motion

1. An object starts moving on a straight track at time $t = 0$. A student is asked to plot the distance covered by the object as a function of time. He makes the following graph.

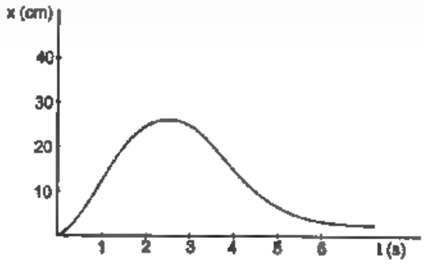


Fig. QB-3

What can you say about the speed of the particle at t = 1 s, 3 s and 5 s? Explain if you find anything wrong in the graph.

2. Look at the proposed v–t graphs for an object moving along a straight line. In each case state whether it is physically possible. If it is not, give reasons.

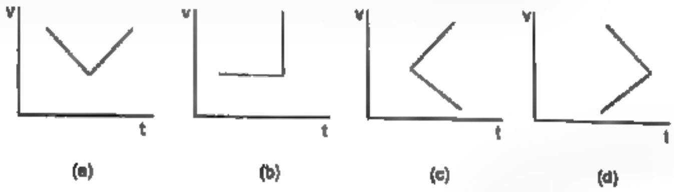


Fig. QB-4

3. Two particles, A and B, move along the same straight line. At $t = 0$, their velocities are zero. Look at their acceleration–time graphs and say whether the statements given below are true or false.

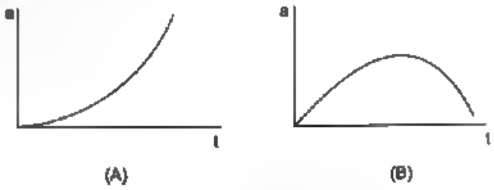


Fig. QB-5

(a) The velocity of each particle is continuously increasing.
(b) The velocity of each particle is continuously decreasing.
(c) The velocity of A increases continuously, but that of B decreases continuously.
(d) Both particles are moving in the same direction.
(e) The velocity of B first increases and then decreases.
(f) One can apply $x = \frac{1}{2}at^2$ for A, but not for B.

[Ans. (a) T, (b) F, (c) F, (d) T, (e) F, (f) F]

Force and Acceleration

4. A person A is sitting on a chair placed on a rough surface. Another person B pushes the chair gently. The chair does not move.

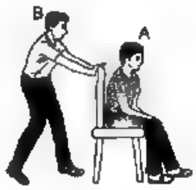


Fig. QB-6

(a) Write all the forces acting on the chair.
(b) Write all the forces acting on A.
(c) Write all the forces acting on B.
(d) Write as many action–reaction pairs as you can.

5. The velocity–time graph of a particle moving eastwards is shown in the figure. For each of the time intervals $t_1 - t_2$, $t_2 - t_3$ and $t_3 - t_4$ write the direction of the resultant force acting on it.

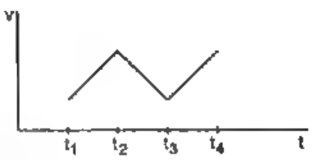


Fig. QB-7

6. A particle moves along a straight line. Its position x as a function of time is shown in the figure.

Fig. QB-8

(a) Is there a nonzero resultant force acting on the particle before time t_1?
(b) Is there a resultant force after time t_1 but before t_2?
(c) Is there a nonzero resultant force at any time in the interval 0 to t_2?

7. Explain why holding a bottle full of water is easier when it is immersed in water than when it is in air.

Work, Energy and Power

8. A ball is thrown up. It reaches a maximum height and then returns. Which of the following have the same values during the upward and downward motions?
   (a) Acceleration
   (b) Force of gravity
   (c) Displacement
   (d) Work done by gravity

Sound

9. Given below are the air density–distance graphs for two different sounds in two different regions. Say whether the amplitude, intensity, wavelength and frequency in A is more than, equal to or less than that in B.

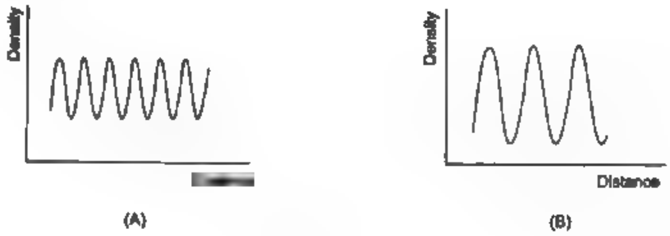


Fig. Q8-9

Objective Questions

I. *Pick the correct option.*

Describing Motion

1. Which statement is correct?
   (a) The earth moves and the sun is at rest.
   (b) The sun moves and the earth is at rest.
   (c) The sun moves with respect to the earth and the earth moves with respect to the sun.
   (d) Both the sun and the earth are at rest.
2. Two trains are moving at speeds 80 km/h and 100 km/h on parallel tracks in the same direction. From this information, one can say that
   (a) the separation between the trains must be increasing
   (b) the separation between the trains must be decreasing
   (c) the separation between the trains remains the same
   (d) the separation between the trains must be changing
3. A particle is moving along a straight line.
   (a) If the acceleration of the particle is more than that of another object, the particle is moving faster.
   (b) If the acceleration is positive, the speed must be increasing.
   (c) If the acceleration and the velocity have the same sign, the speed must be increasing.
   (d) If the acceleration is zero, the velocity is constant but the speed may be changing.
4. The distance–time graph of a particle moving along a curved path is a straight line.
   (a) The speed of the particle is constant.
   (b) The velocity of the particle is constant.
   (c) The acceleration of the particle is zero.
   (d) The displacement–time graph is a straight line.

5. Look at the given distance–time graph of an object.

Fig. QB-10

(a) The speed of the object is increasing continuously.
(b) The speed of the object is decreasing continuously.
(c) The speed of the object is constant.
(d) The acceleration of the object is zero.

6. A particle moves along a straight line. Its velocity and speed are denoted by v_1 and v_2. The quantity $\frac{v_1}{v_2}$

(a) must be equal to 1
(b) must be equal to –1
(c) can be any positive number
(d) must be either +1 or –1

Force and Acceleration

7. Three forces act on a particle and keep it at rest. It is possible to have
(a) all the forces acting eastwards
(b) two forces acting eastwards and one acting northwards
(c) one force acting eastwards, one westwards and one southwards
(d) one force acting eastwards and two forces westwards

8. Which of the following is a contact force?
(a) Frictional force
(b) Gravitational force
(c) Magnetic force
(d) Electric repulsion force

9. Normal force always acts
(a) in the vertically upward direction
(b) in the vertically downward direction
(c) perpendicular to the surface of contact
(d) parallel to the surfaces of contact

10. A heavy ball is dropped from a height on a pile of sand. After falling on the pile, the ball moves some distance in the sand and then stops. During its motion in the sand, the force F_w on the ball due to its weight and the force F_s due to sand
(a) are equal in magnitude
(b) are in the same direction
(c) are such that $F_w > F_s$
(d) are such that $F_w < F_s$

11. Consider a cylindrical rod immersed in water in a vertical position. The force by the water on the rod is
(a) downwards everywhere
(b) upwards everywhere
(c) downwards on the top surface, upwards on the bottom surface
(d) upwards on the top surface, downwards on the bottom surface

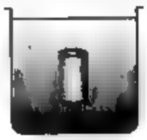


Fig. QB-11

Work, Energy and Power

12. To open the door of a room, you push on the handle, and to close it you pull on the handle. The work done by you on the handle is
    (a) positive in both cases
    (b) negative in both cases
    (c) positive while opening the door but negative while closing it
    (d) positive while closing the door but negative while opening it

Sound

13. A sound wave is generated due to a vibrating tuning fork. The separation between a layer of maximum compression and the next layer of normal density is
    (a) λ (b) $\frac{\lambda}{2}$
    (c) $\frac{3\lambda}{4}$ (d) $\frac{\lambda}{4}$

14. The amplitude of density variation corresponding to a sound wave is 1 milligram/metre^3. The maximum difference in the density between different layers of air will be
    (a) $1\ mg/m^3$ (b) $2\ mg/m^3$
    (c) $\frac{1}{2}\ mg/m^3$ (d) $\frac{1}{4}\ mg/m^3$

II. *Mark the statements True (T) or False (F).*

Describing Motion

1. The magnitude of the velocity of an object is equal to its speed at that instant.
2. The magnitude of the speed of an object is equal to its velocity at that instant.
3. The velocity of a particle at an instant is –4 m/s. The particle must be moving along a straight line.
4. The velocity of a particle at an instant is 4 m/s. The particle must be moving along a straight line.
5. The position–time graph of a particle moving along a straight line is as shown in the figure. Its speed is constant.

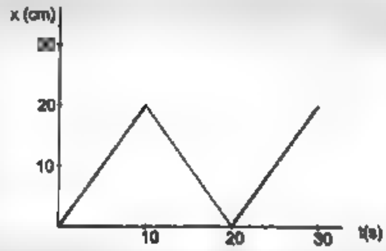


Fig. Q8-12

6. A particle moves along a straight line. Its speed and velocity are both given by +3 m/s at an instant. The particle is moving along the positive direction of the line.

Force and Acceleration

7. If the acceleration of an object is not zero, its speed must be changing.
8. If the acceleration of an object is zero for a period, it cannot change its direction of motion during this period.
9. Accelerations of two objects are 2.0 m/s^2 and 4.0 m/s^2 at an instant. It is possible that their speeds are equal at this instant.
10. When a force acts on a body, its speed necessarily changes.

11. Forces can only be applied by living bodies.
12. When you hold a bucket of water, you apply an upward force on the bucket, which is equal to the combined weight of the bucket and the water.
13. In action–reaction pair of forces, action force acts before the reaction force.
14. Equal forces acting on two different bodies always produce the same acceleration.
15. When a steel rod is immersed in water, the force exerted by the water on the rod is equal to the weight of the rod.
16. When a plastic rod floats in water, the force exerted by the water on the rod is equal to the weight of the rod.
17. When a plastic rod is completely immersed in water, the force exerted by the water on the rod is equal to the weight of the rod.

Work, Energy and Power

18. According to the principle of conservation of energy, if the kinetic energy of an object is decreased by 50%, its potential energy will be increased by 50%.

Sound

19. A stone hits a glass plate, which breaks. When this happens, the density of the air around the plate changes.
20. The pitch of sound is measured in hertz.
21. A 2-kHz sound will spread less around the edges of a cardboard as compared to a 1-kHz sound.
22. A louder sound bends more around the edges than a less-loud sound of the same frequency.

Numericals

Measurements

1. The allowed limit of arsenic in drinking water is 0.00005 grams per litre. Convert this into SI unit.
2. The radius of a hydrogen atom is 5.3×10^{-11} m. Convert this into a unit with a standard prefix before metre so that the numerical value is between 1 and 1000.
3. What is the radius of the earth in megametres?
4. Convert 1 yard into metres.
5. Convert 100 square yards into square metres.

Describing Motion

6. What is the distance moved by a particle moving along the x-axis, if it starts from $x=-7$ m, goes up to $x=-12$ m, turns and stops at $x=+3$ m?
7. In the above problem what is the net displacement?
8. Delhi and Chicago are almost diametrically opposite on the earth. If someone travels from Chicago to Delhi, what will be the magnitude of his displacement?
9. In the hare and tortoise race, the tortoise ran at a uniform speed of 10 cm/s and reached the finishing line in 72 minutes. The hare ran at 1 m/s for the first 6 minutes and then slept for 65 minutes. He then ran at 1 m/s to reach the finishing line.
   (a) When the hare went to sleep, what was the position of the tortoise? What was the position of the hare at that time?
   (b) After how much time after the start did the tortoise cross the hare?
   (c) When the tortoise reached the finishing line, where was the hare?
   (d) By how much time did the hare lose the race?
   (e) Plot the position versus time plots on the same graph for the hare and the tortoise.
10. Trains A and B are moving on parallel tracks in the opposite directions. A is moving at 80 km/h and B is moving at 100 km/h. At $t=0$, they are 60 km apart. Plot the 'separation' between the trains (front parts of the engines) as a function of time from $t=0$ to $t=60$ min.

11. The velocity of a particle moving along a straight line varies with time as shown in the figure. Find the total distance covered and the total displacement between 0 and 20 seconds.

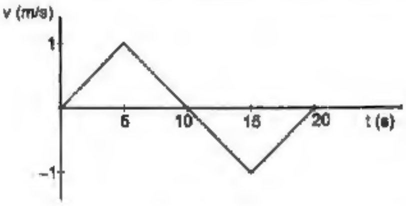


Fig. Q8-13

Force and Acceleration

12. The length of the outer edge of a cubical vessel is 10 cm and the length of its inner edge is 9.8 cm. If it is immersed in water just up to its full height without allowing water to enter the vessel, how much volume of water will it displace?
13. The mass of a cricket ball is 160 g. If a fast bowler bowls with the ball at a speed of 144 km/h, what is the linear momentum of the ball in SI units?
14. A bicycle of mass 10 kg is moving on a road at a speed of 14.4 km/h. It reaches a downward slope, and its speed increases to 21.6 km/h in 10 seconds. Find the resultant force on the bicycle during this period. If the rider weighs 50 kg, what is the resultant force on the rider during this period?

Gravitation

15. Find the gravitational force that a 1-kg salt packet in your kitchen exerts on the earth.

Sound

16. A tuning fork vibrating at a frequency of 340 Hz is placed 5 cm from a human ear. The speed of sound in air is 340 m/s. At a particular instant the density of air is maximum adjacent to the prong. Will all the air between the prong and the air be compressed at this time or will there be a series of compressed and rarefied layers in this region? Neglect any reflection from any object.
17. A sound wave travels in water at the speed of 1500 m/s. The minimum separation between two layers of the same density is 75 cm. Find the frequency of the sound.
18. The audible range for an average human being is 20 Hz–20 kHz. If the speed of sound is 350 m/s, what is the wavelength range of audible sound?

• ANSWERS •

Objective Questions

I.	1. (c)	2. (d)	3. (c)	4. (a)	5. (d)	6. (d)
	7. (d)	8. (a)	9. (c)	10. (d)	11. (c)	12. (a)
	13. (d)	14. (b)				
II.	1. (T)	2. (F)	3. (T)	4. (F)	5. (T)	6. (T)
	7. (F)	8. (T)	9. (T)	10. (F)	11. (F)	12. (T)
	13. (F)	14. (F)	15. (F)	16. (T)	17. (F)	18. (F)
	19. (T)	20. (F)	21. (T)	22. (F)		

Numericals

1. 5×10^{-5} kg/m^3 2. 53 picometres 3. 6.4 4. 0.9144 m 5. 84.02736 m^2 6. 20 m 7. 10 m

8. 12,800 km

9. (a) In 6 minutes, the hare ran 1 m/s × (6 × 60) s = 360 m, and the tortoise ran 0.1 m/s × (6 × 60) s = 36 m.

(b) To cover 360 m, the tortoise took 360 m/0.1 m/s = 3600 s = 60 min. So, it crossed the hare at t = 60 min.

(c) The tortoise reached the finishing line in 72 min. In these 72 min, the hare slept for 65 min and ran during the rest of the time (7 min). The position of the hare after 72 min was (7 × 60) s × 1 m/s = 420 m from the starting point.

(d) The total distance of the race was 72 min × 10 cm/s = (72 × 60) s × 0.1 m/s = 432 m. In 72 min the hare had covered 420 m, and he covered the remaining 12 m in $\frac{12 \text{ m}}{1 \text{ m/s}}$ = 12 s. So, he lost by 12 seconds.

(e)

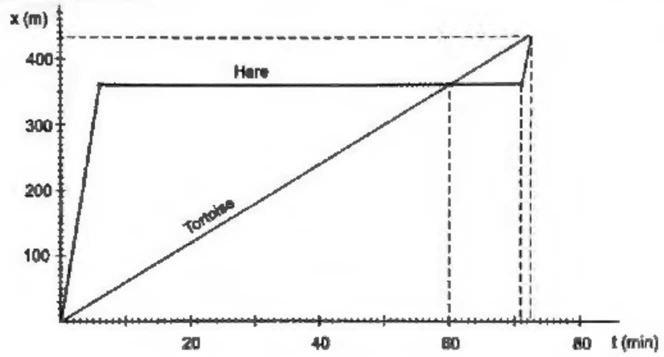


Fig. QB-14

10. Initially the separation between the trains will decrease at the rate 180 km/h. So, the front parts of the engines will cross at t = 60 km/(180 km/h) = 20 min. After that, the separation will increase at the rate of 180 km/h. In the next 40 min, the separation will be s = (180 km/h) × 40 min = (180 km/h) × (40/60) h = 120 km.

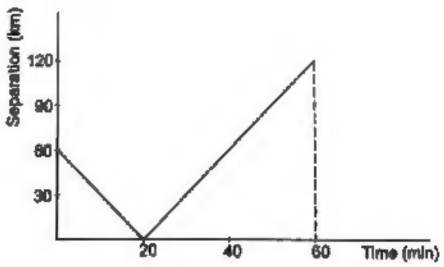


Fig. QB-15

11. 5 m, zero 12. 1000 cm^3

13. 160 g = 0.16 kg, 144 km/h = (144 × 10^3) m/3600 s = 40 m/s.
p = 0.16 kg × 40 m/s = 6.4 kg m/s.

14. 14.4 km/h = 4 m/s, 21.6 km/h = 6 m/s, $a = \frac{(6-4) \text{ m/s}}{10 \text{ s}} = 0.2 \text{ m/s}^2$.

F_{cycle} = 10 kg × 0.2 m/s^2 = 2 N, F_{rider} = 50 kg × 0.2 m/s^2 = 10 N.

15. 9.8 N

16. The wavelength is $\lambda = \frac{v}{\nu} = 1$ m. The distance between a maximum-density layer and the next normal-density layer is $\frac{\lambda}{4} = 25$ cm. As the distance between the prong and the ear is only 5 cm, all the air between them will be compressed at this instant.

17. $\lambda = 150$ cm, $\nu = \frac{v}{\lambda} = \frac{1500 \text{ m/s}}{1.5 \text{ m}} = 100$ Hz.

18. $\lambda_1 = \frac{v}{\nu} = \frac{350 \text{ m/s}}{20/\text{s}} = 17.5$ m, $\lambda_2 = \frac{350 \text{ m/s}}{20 \times 10^3/\text{s}} = 17.5$ mm.

❖